



OXAND

Optimizing Infrastructure Solutions



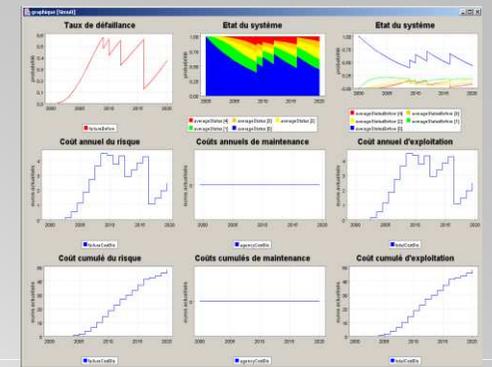
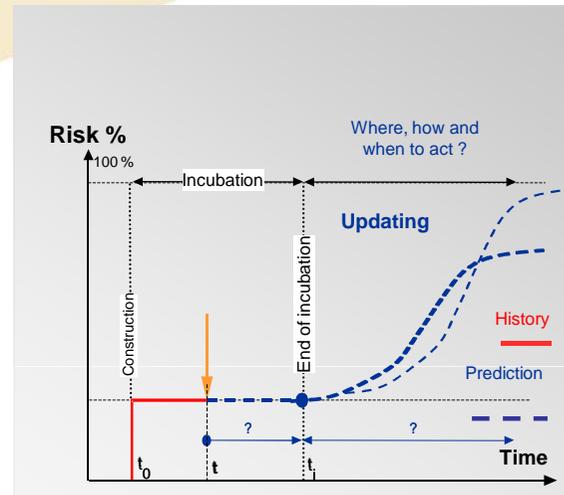
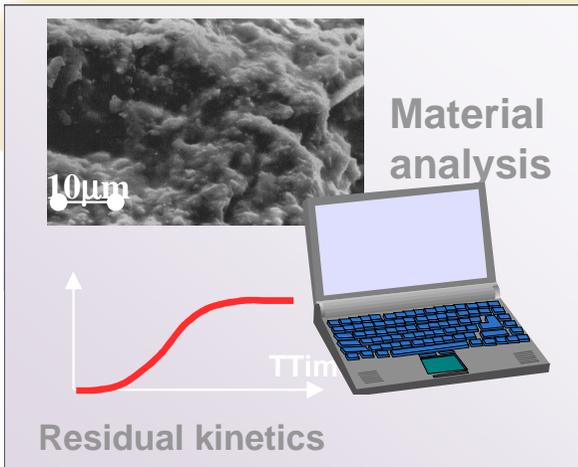
Effects of carbonation on the lifetime of concrete structures and updating by Bayesian networks

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DECISION MAKING: INTEGRATION OF KNOWLEDGE



Data Acquisition

Data Treatment

Risk Management and Updating

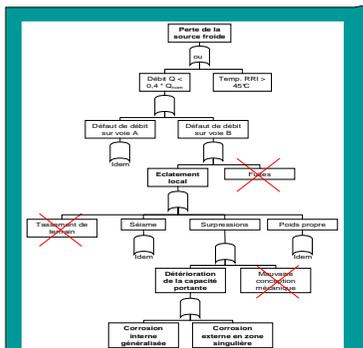


Simulations

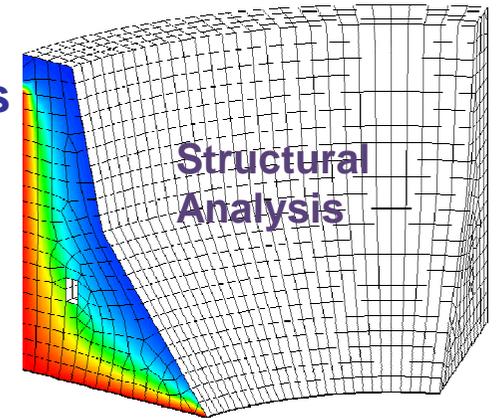
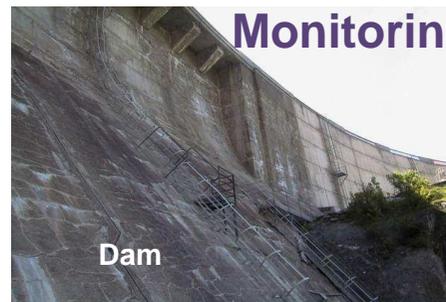
Quantification

Monitoring

Structural Analysis

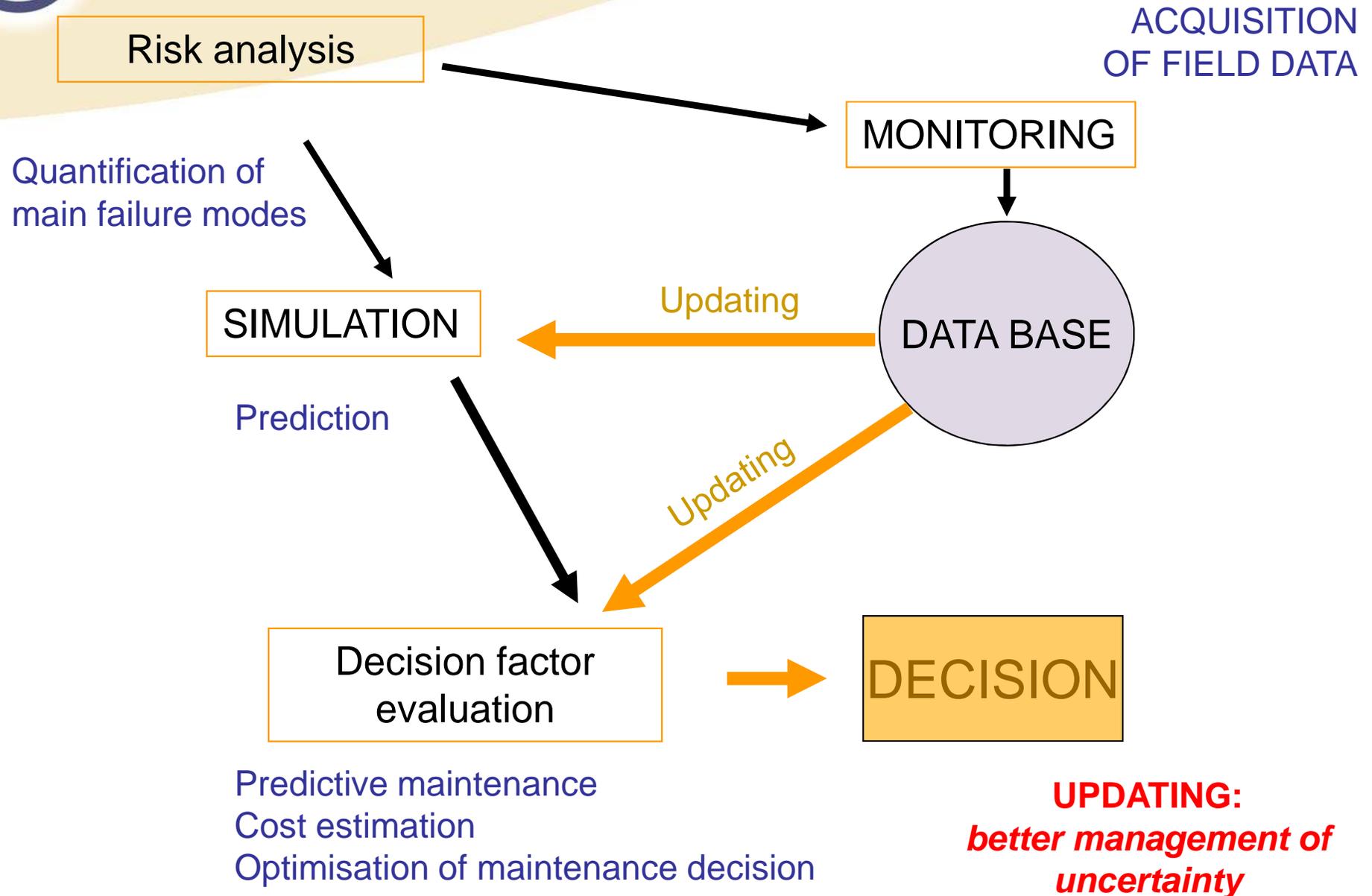


Risk Analysis





Risk management associated to ageing





BAYESIAN APPROACH

Basic principles

Factory: 3 machines : M1, M2, M3
Production: white and red parts
(Color C: discrete variable)

1	2	3	3
1	2	3	3
1	2	3	3
1	2	3	3

M1 M2 M3

Probabilities to have white or red parts:

$$P(C=\text{white}) = \frac{1}{2}$$

$$P(C=\text{red}) = \frac{1}{2}$$

- Conditional probabilities :

$$P(C=\text{red} | M1) = 1$$

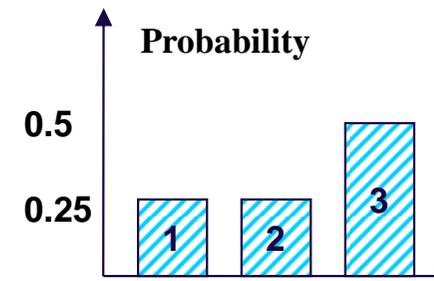
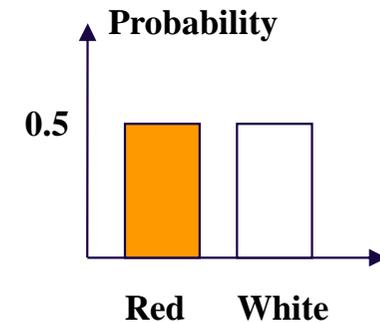
$$P(C=\text{red} | M2) = \frac{1}{2}$$

$$P(C=\text{red} | M3) = \frac{1}{4}$$

- A priori : origin of the part

$$P(M1) = P(M2) = \frac{1}{4}$$

$$P(M3) = \frac{1}{2}$$





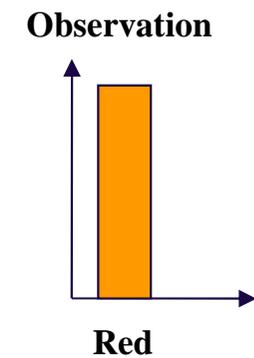
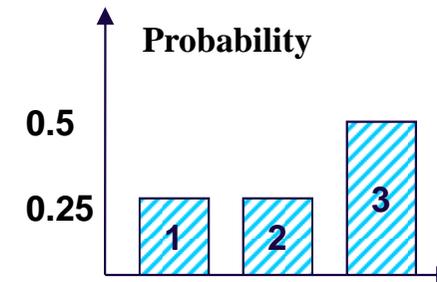
BAYESIAN APPROACH

Basic principles

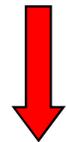
- A priori: origin of the part

$$P(M1) = P(M2) = \frac{1}{4} \quad P(M3) = \frac{1}{2}$$

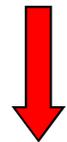
- Observation : red piece picked



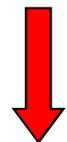
Prior



Obs.



Updating process

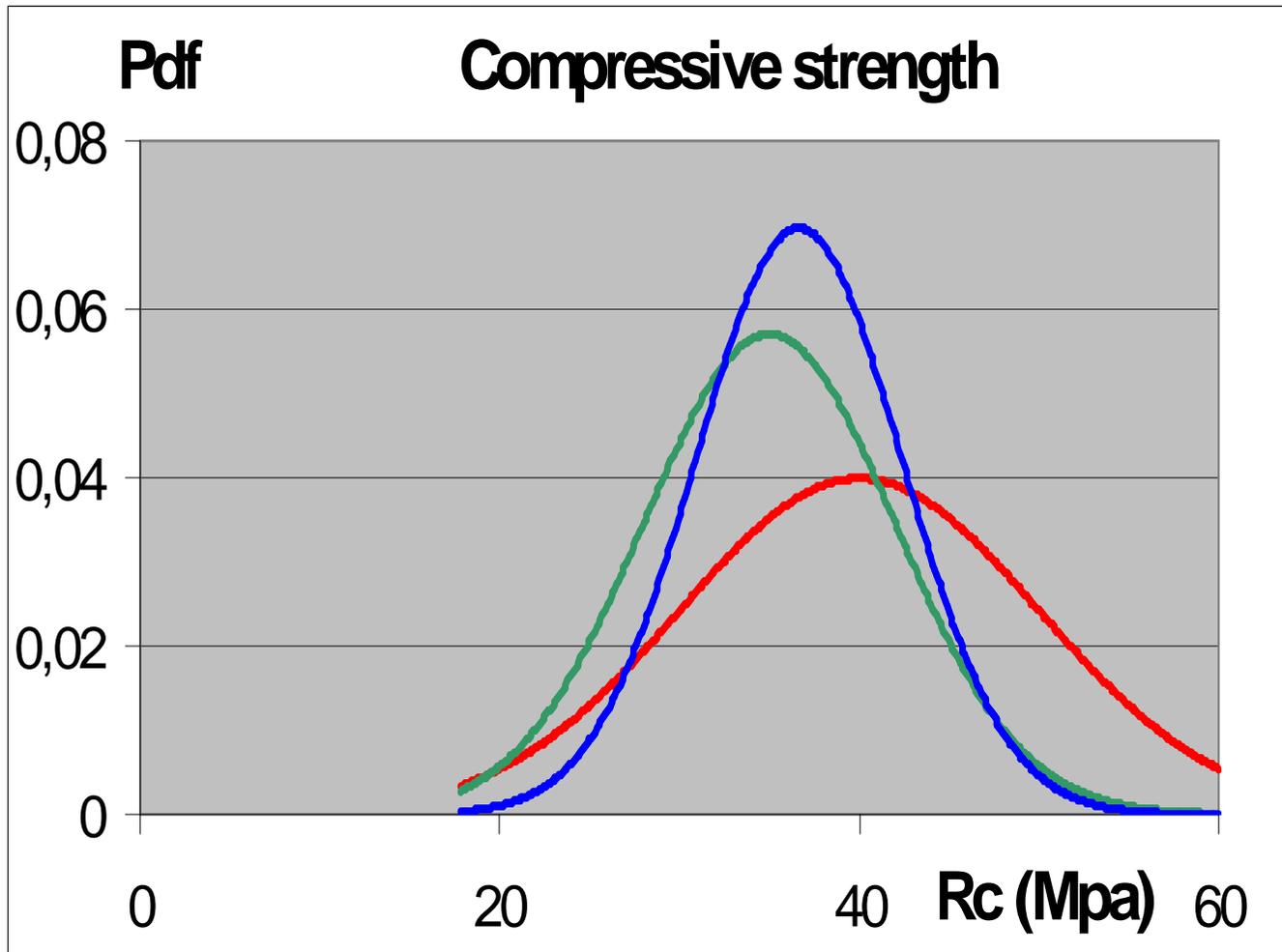


Posterior



Updating of a continuous parameter

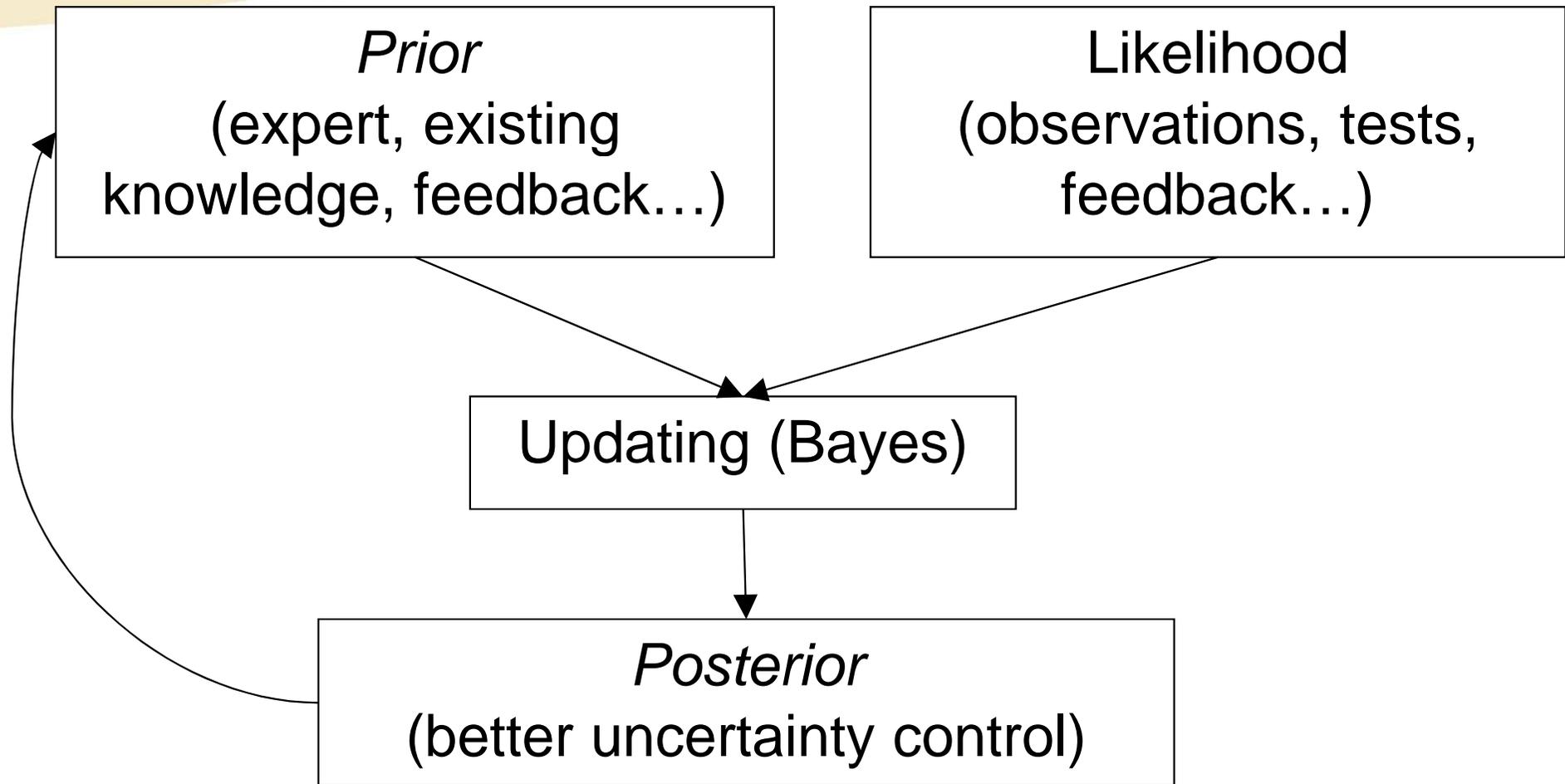
Example with Normal distributions



- 1. *A priori law:***
 $N(40, 10)$
- 2. *Observation:***
 $N(35, 7)$
- 3. *A posteriori:***
 $N(36.6, 5.7)$



Bayesian approach





Bayesian networks

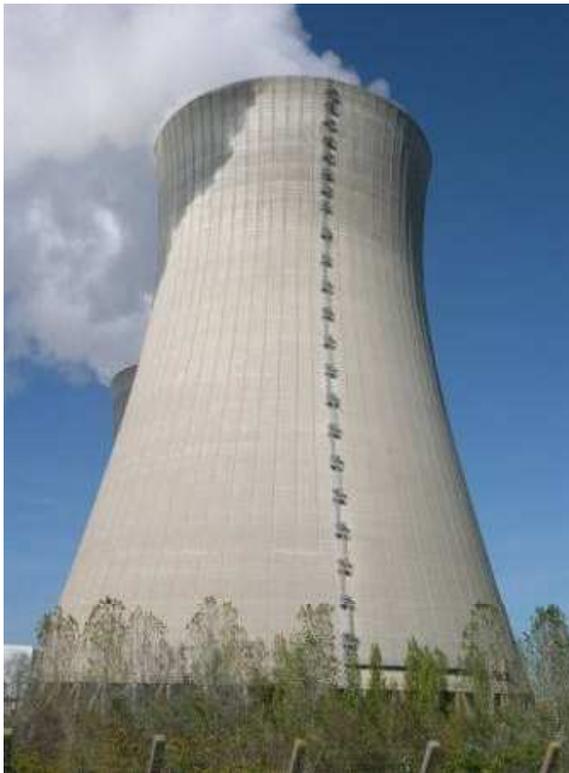
- **Simple updating :**
evolution of one single parameter

=> Bayes' theorem
(analytical solution for some cases)
- **Degradation modeling function of numerous uncertain parameters**
=> Bayesian networks
(numerical simulation)

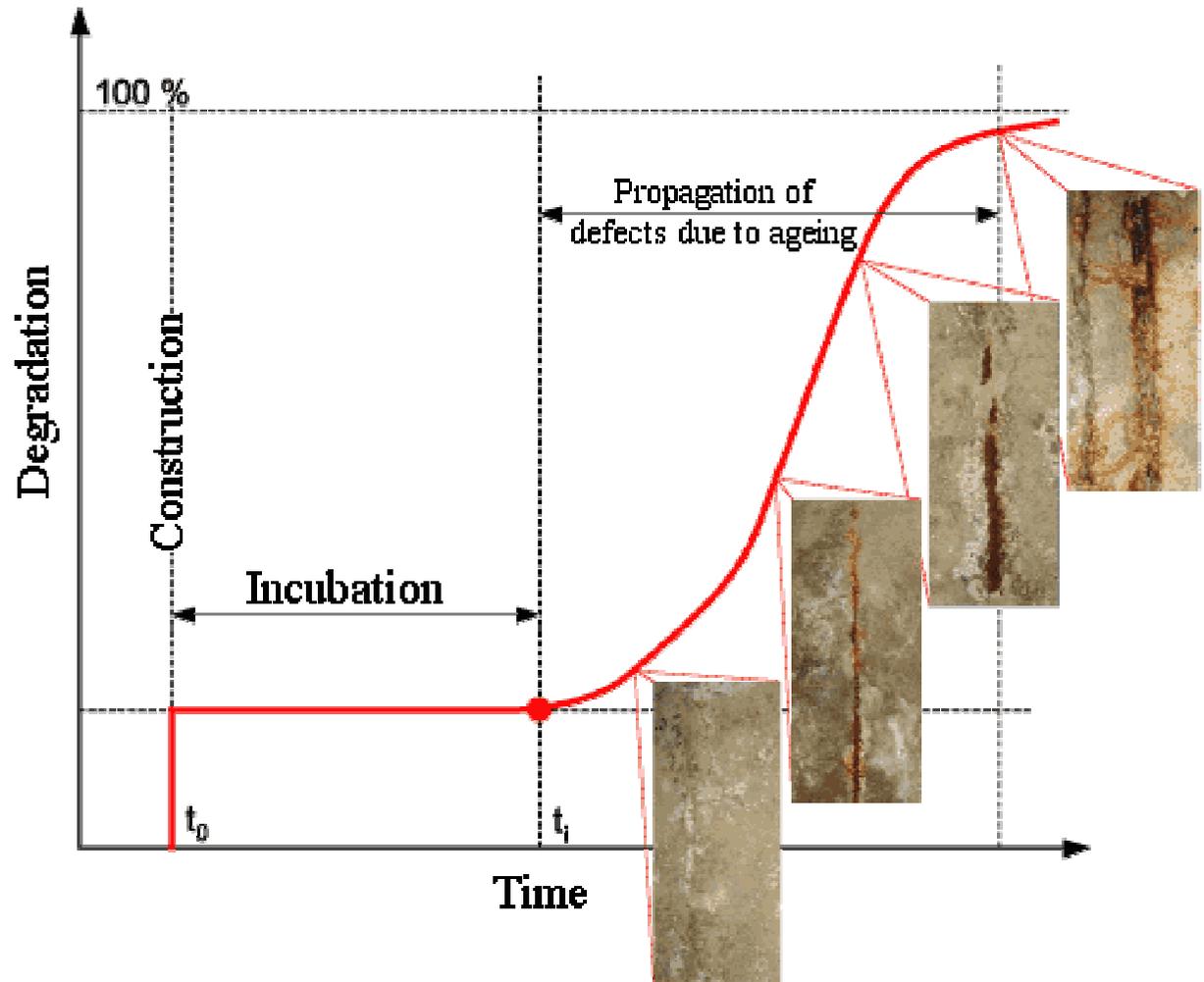


Life time of concrete structure

Concrete structure



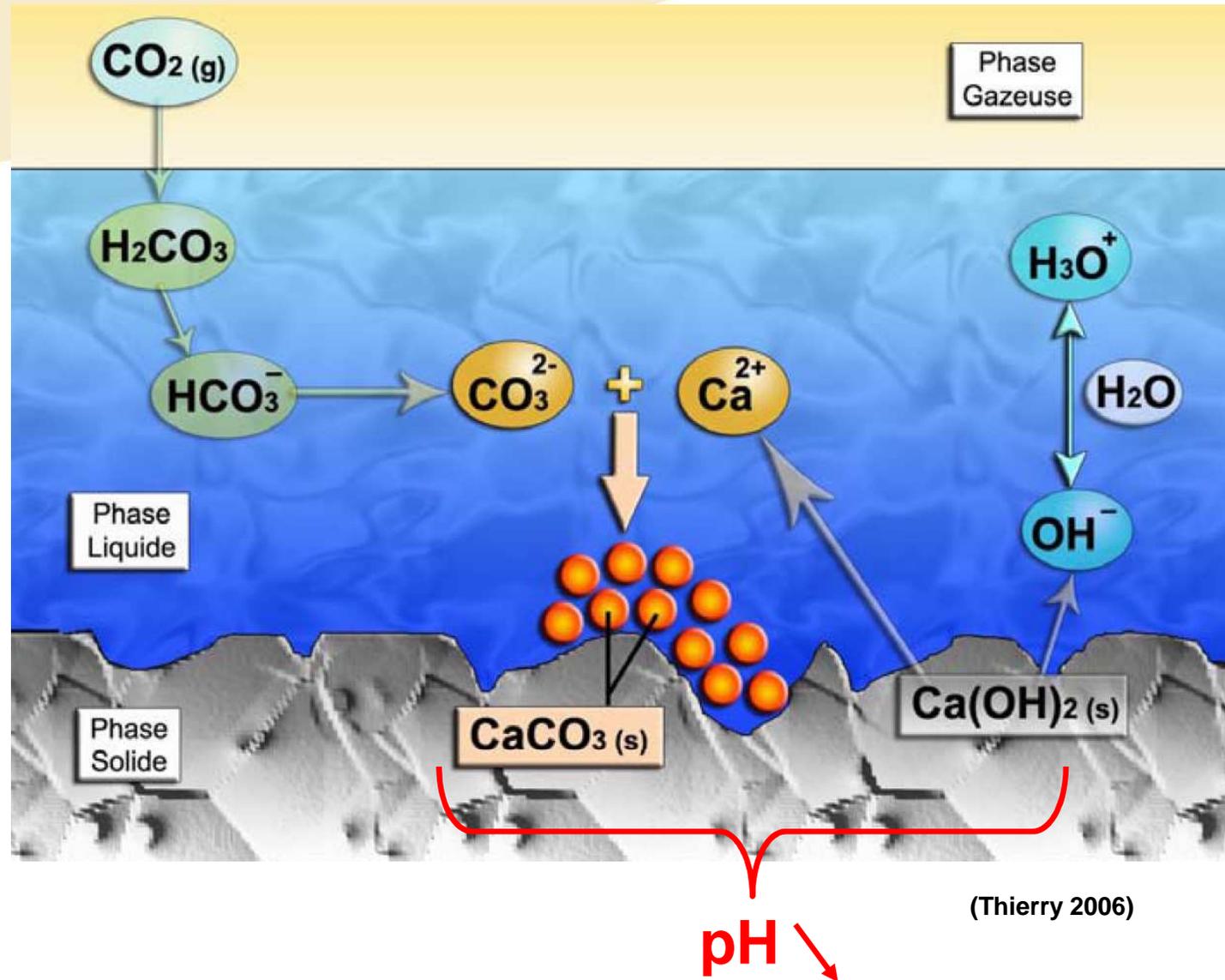
Concrete degradation vs time



Life time assessment: important stake for structure owner



Carbonation of concrete

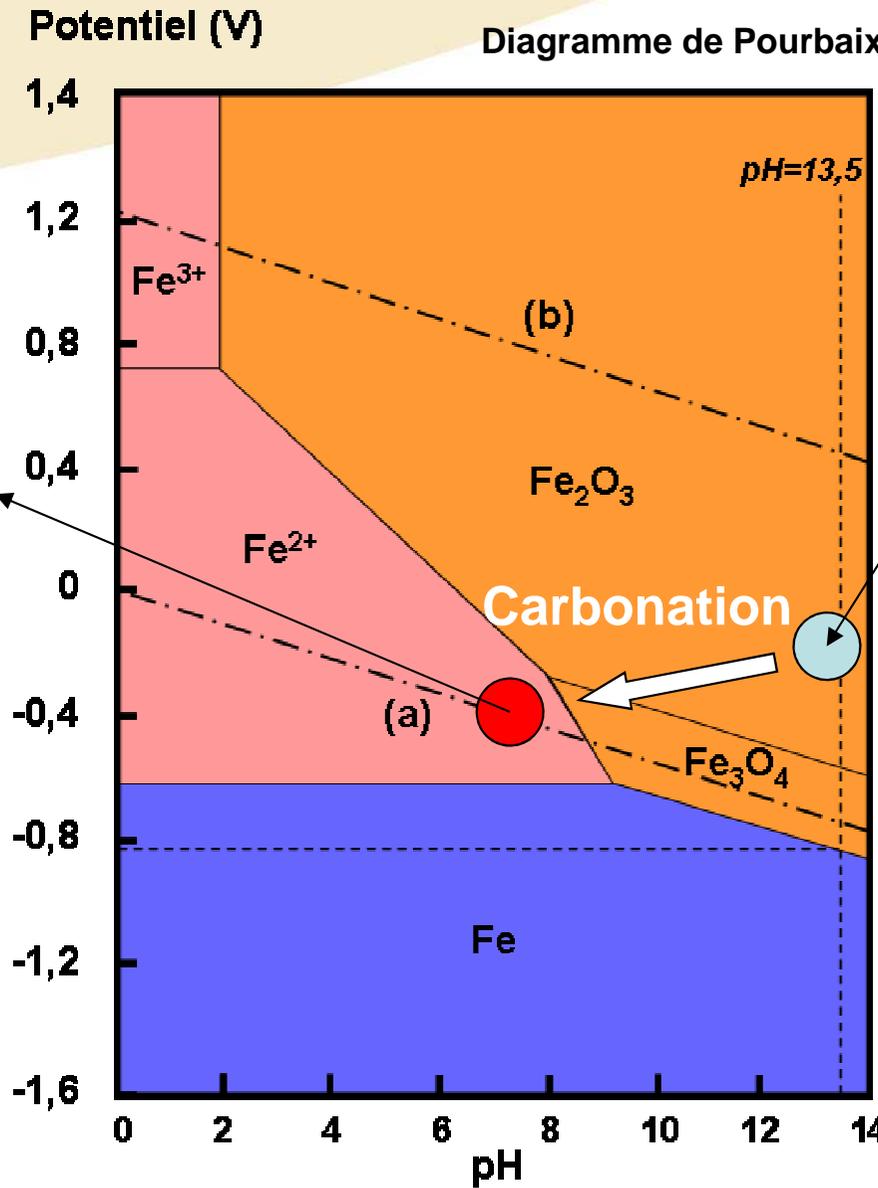


Strongly depends on transport properties of concrete (diffusivity, porosity)



Corrosion of steel in concrete

Diagramme de Pourbaix du système Fe-H₂O à 25°C



Carbonated concrete
($\text{pH}=7$; $E=-400$ mV)

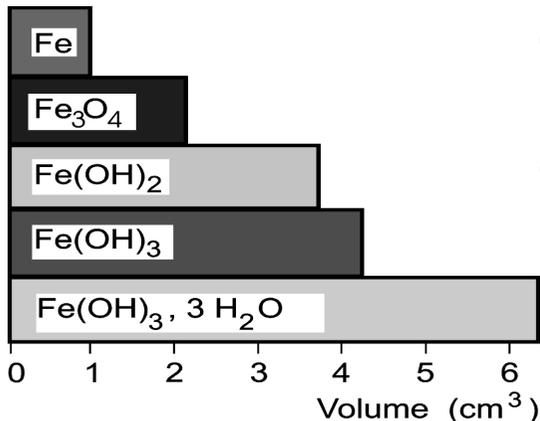


CORROSION



Sound concrete
($\text{pH}=13,5$; $E=-200$ mV)

Volume increase of steel
oxydation products



**Mechanical stresses leading to
concrete cracking**

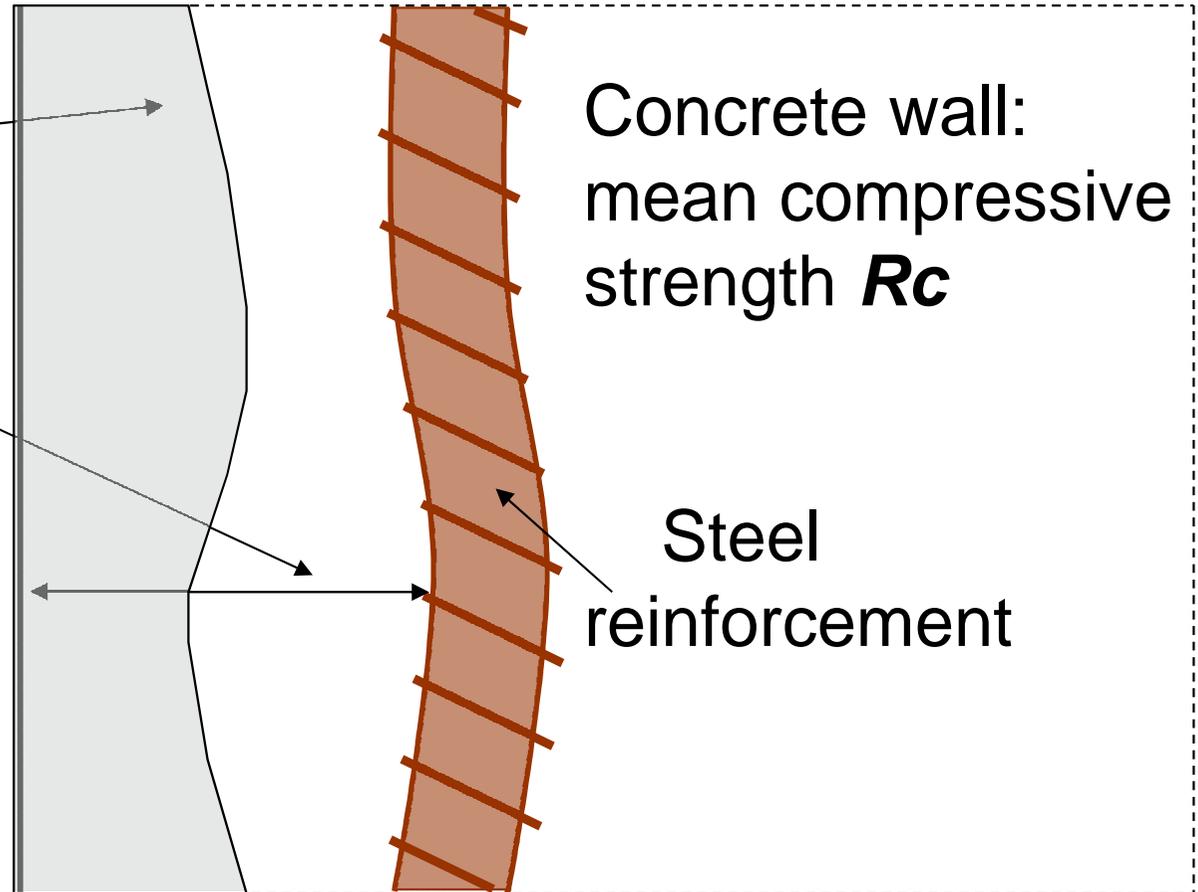


Corrosion risk for concrete structures

Carbonation depth
 $X = f(R_c, RH, t, \dots)$

d : concrete cover

Environment:
relative humidity
 RH



Limit state: failure when $x \geq d$ (corrosion starts)



Carbonatation models for concrete

- **Engineering models:**

$$x = \gamma f(RH) k(Rc) \sqrt{t} \quad (\text{Petre Lazar 2001})$$

- with:
- RH: relative humidity
 - Rc: compressive strength of concrete
 - γ : exposition coefficient to CO₂
 - t: time

and:

$$\begin{cases} f(RH) = -3.5833RH^2 + 3.4833RH + 0.2 \\ k(Rc) = \sqrt{365} \left(\frac{1}{2.1\sqrt{Rc}} - 0.06 \right) \end{cases}$$

Carbonation rate mainly depends on Rc: influence of cement type ?



Carbonatation models for concrete

- **Engineering models related to physical parameters:**

Carbonation rate related to the amount of cement hydrates likely to be carbonated (C-S-H but also Portlandite, Ettringite and Aluminates) and CO₂ pressure

$$x_c(t) = \sqrt{\frac{2 \cdot \text{err1} \left(\frac{D_{CO_2}^0}{Q_1} \right)_{ref} Q_1 P_0 t}{RT \left(1 + \beta C_2 \left(\frac{P_0}{P_{atm}} \right)^n \right) \left(\frac{\varphi_p C_2}{1+n} \left(\frac{P_0}{P_{atm}} \right)^n + Q_1 \right)}}$$

(Hyvert et al 2010,
Mai-Nhu et al 2012)

Where:

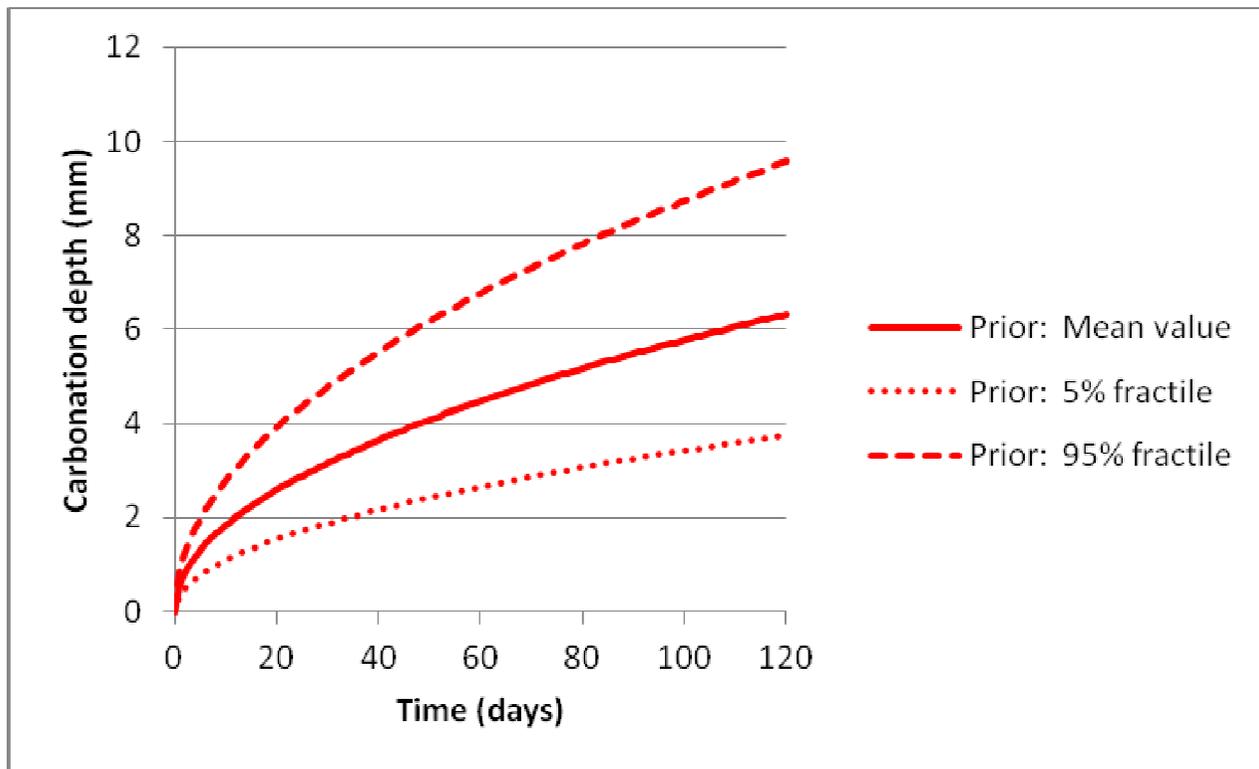
$$\text{err1} = \frac{\left(\frac{D_{CO_2}^0}{Q_1} \right)_{\text{mod èle}}}{\left(\frac{D_{CO_2}^0}{Q_1} \right)_{ref}} = \frac{\frac{(x_c^{\text{exp}})^2 RT}{2 P_{Khan} t_{Khan}}}{6,44 \cdot 10^{-13} \left(27,2 e^{-0,032 f_{cm,28}} \right)^2}$$



Prior estimation of carbonation depth

• APPLET's French national project:

- Life time of structures, variabilities and uncertainties related to concrete characteristics
- Compressive strength (CEM I + silica fume): mean value at 28 days : 58.3 MPa
standard dev : 4.3 MPa (CV=7.3 %)
- Accelerated carbonation test (CO_2 pp = 50%)



Uncertainties:

Concrete compress. strength:
LogN (58,3 MPa; 4,3 MPa)

Model error:
LogN (1,11 MPa; 0,52 MPa)



Experimental data

- **APPLET's French national project:**

Carbonation tests at different times (CO₂ pp = 50%)

Age	Carbonation depth (mm)										Mean value (mm)	Standard deviation (mm)	Coefficient of variation
28 j	5,2	6	5,2	2	3,8	4,8	6,2	6	2,2	2	4,34	1,72	39,5 %
90 j	6,5	8,2	5,7	2,7	6	5,5	7,2	6,8	4,3	4,5	5,74	1,59	27,8 %

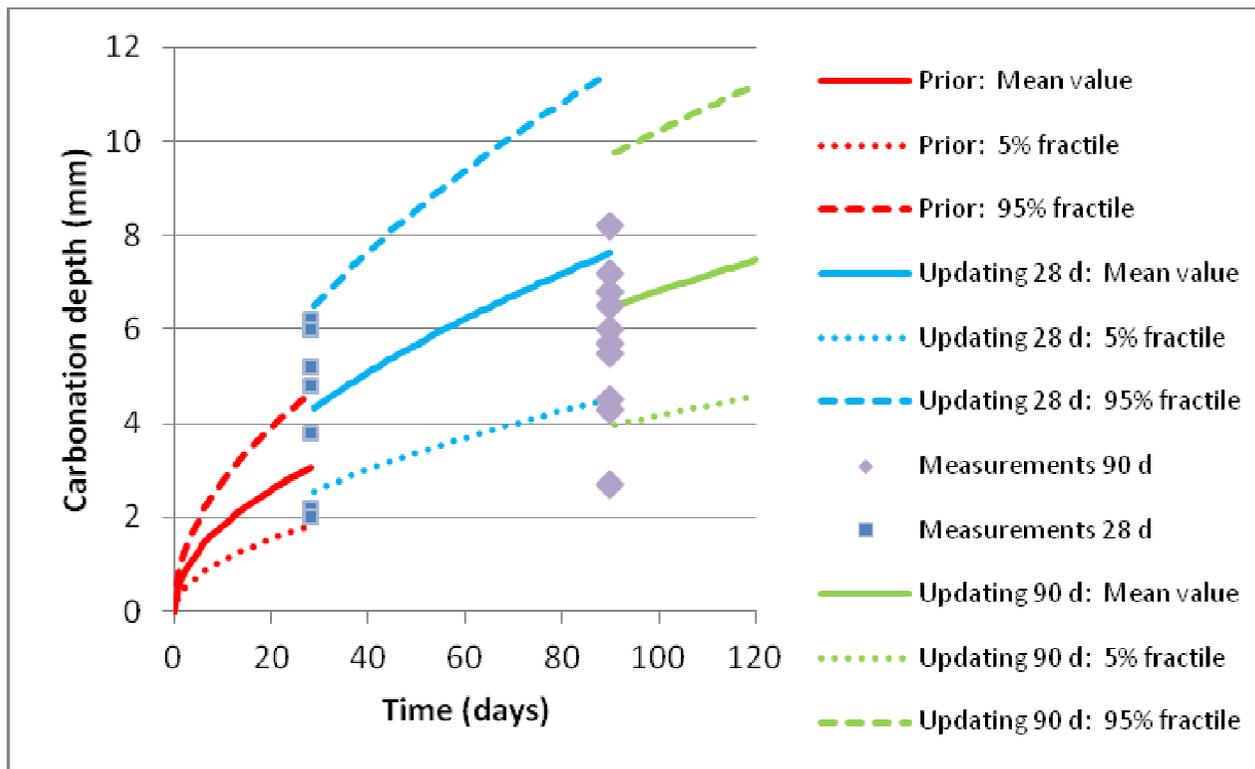
Strong variability: 30 to 40% !



Posterior estimation of carbonation depth

- **APPLET's French national project:**

- Experimental data are used to update the prior estimation of carbonation depth



Model/experimental data:

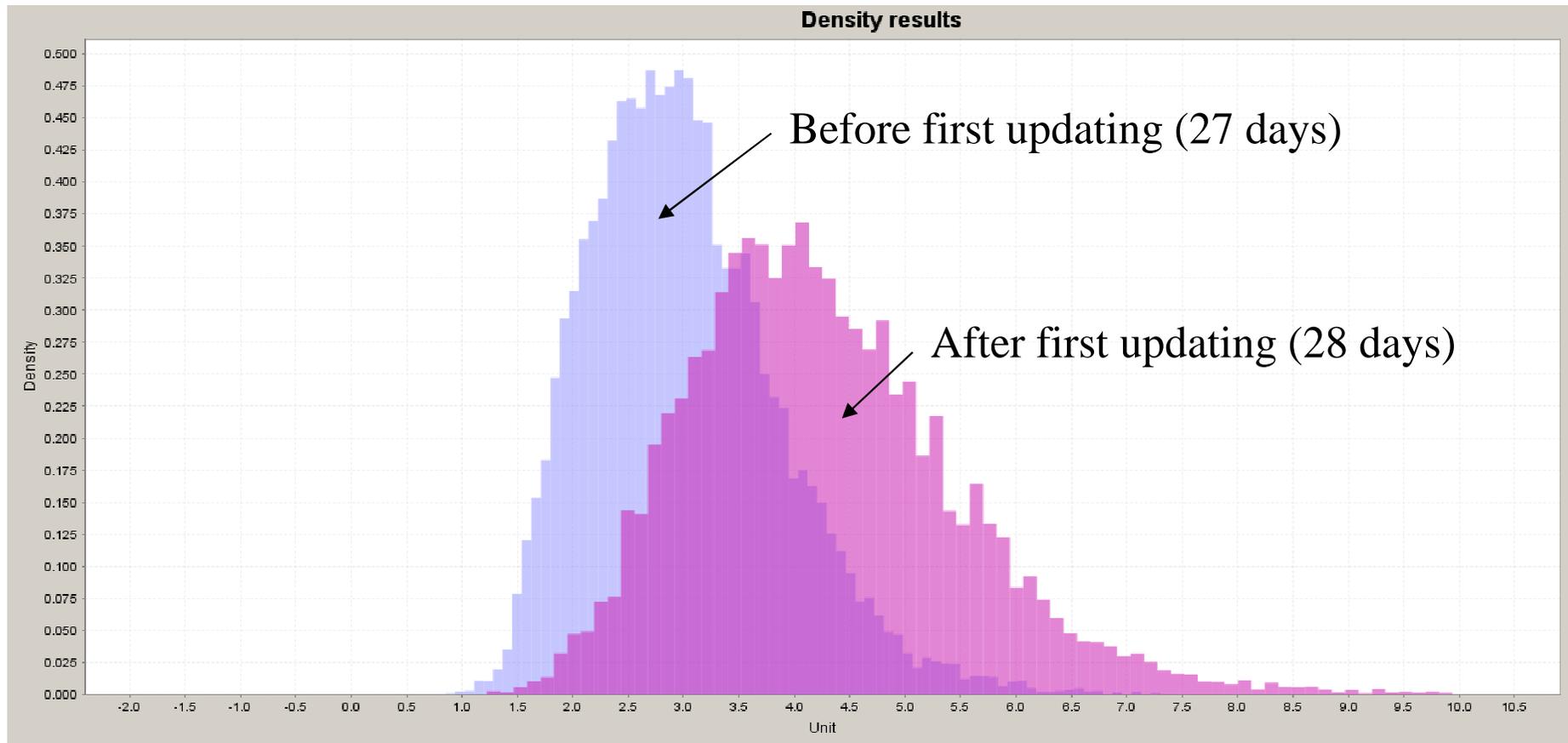
- 28 days: underestimation of carbonation depth
- 90 days: overestimation of carbonation depth
- uncertainty \nearrow then \searrow (CV from 30 to 40 %)



Posterior estimation of carbonation depth

- Evolution of uncertainty

- First updating at 28 days: pdf of carbonation depth

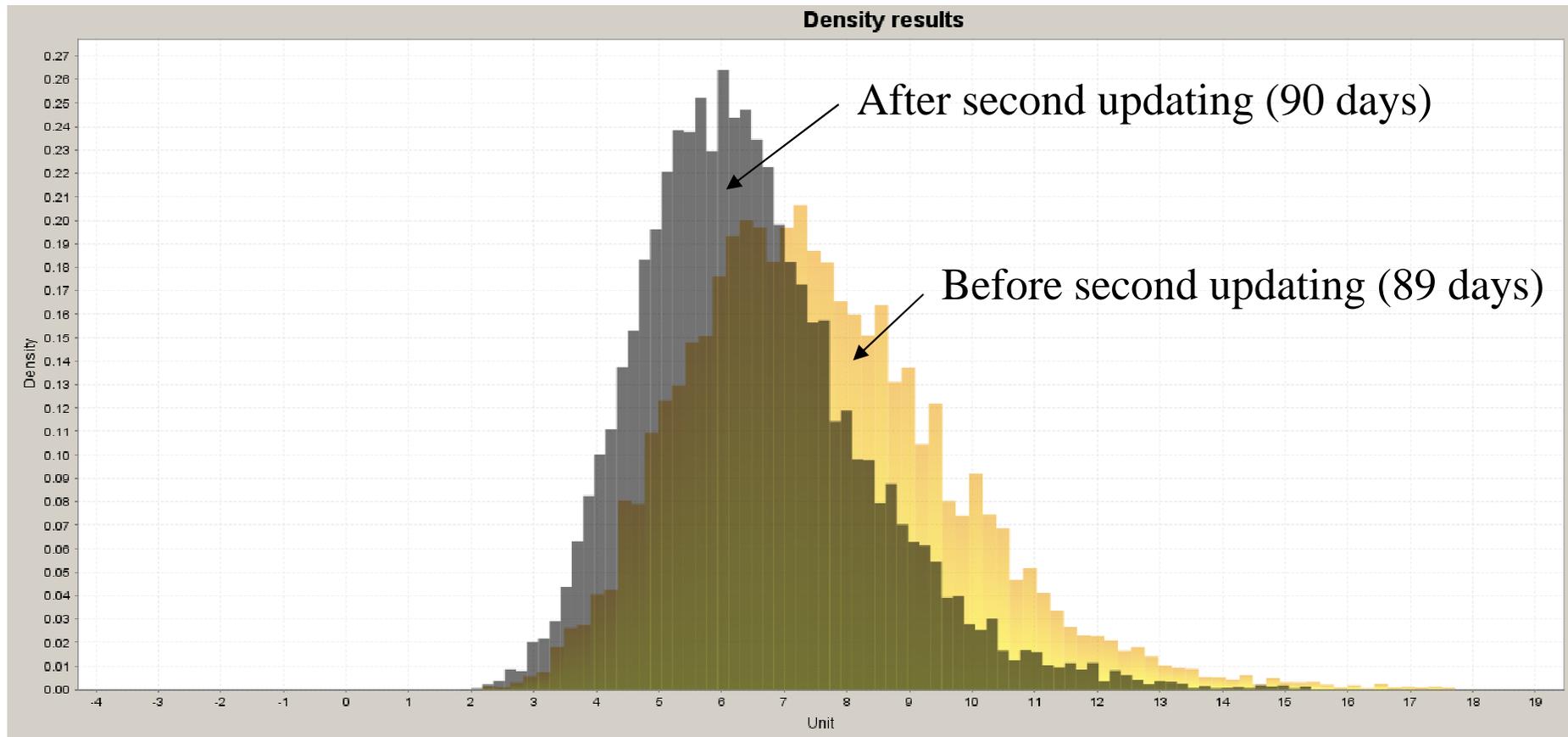




Posterior estimation of carbonation depth

- Evolution of uncertainty

- Second updating at 90 days: pdf of carbonation depth





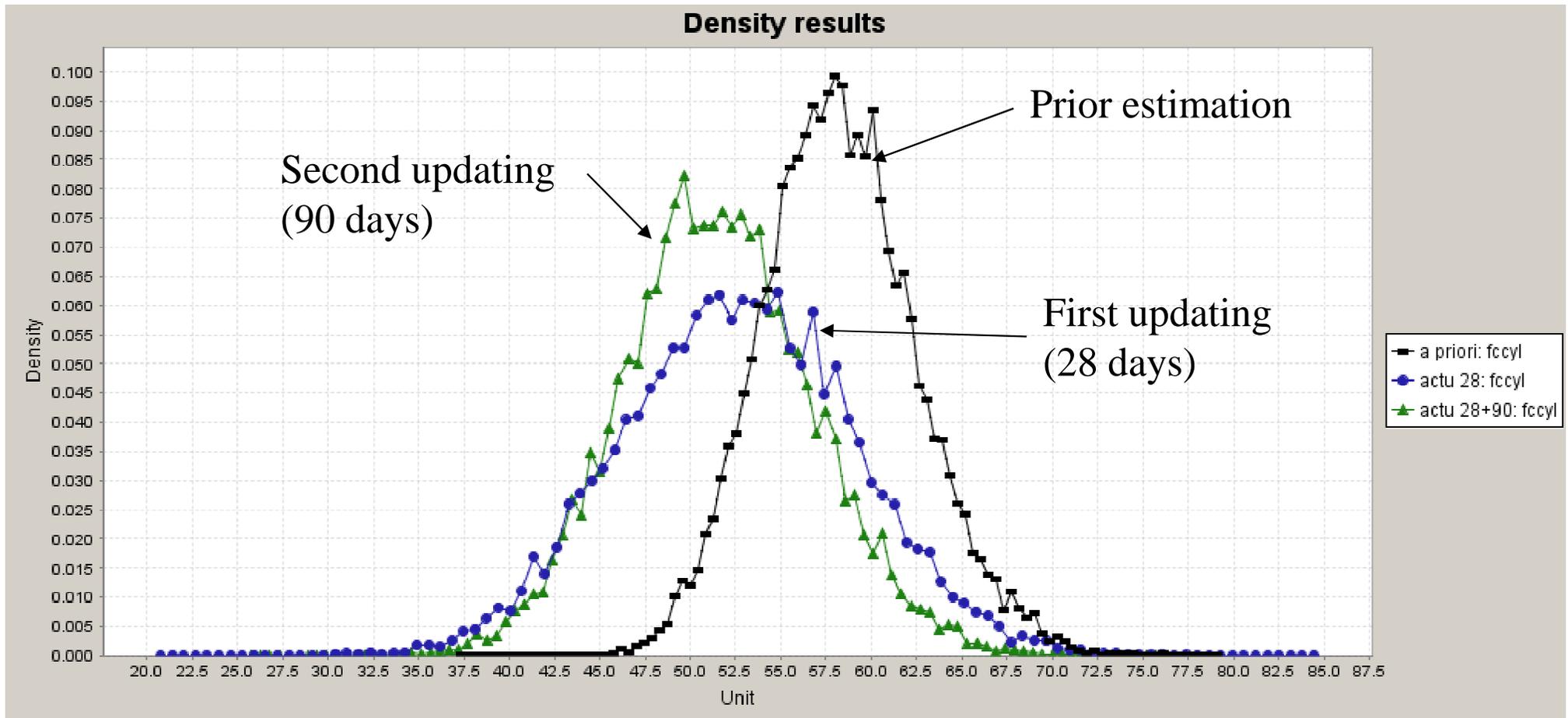
Updating of the random variables

Random variable	Compressive strength (MPa)	Model error
Initial data	Lognormal law: <i>Mean: 58,3</i> <i>Standard deviation: 4,3</i>	Lognormal law: <i>Mean : 1,10</i> <i>Stand. Dev.: 0,5</i>
Prior simulated (Monte Carlo) (400 000 drawings)	<i>Mean: 58,2</i> <i>Stand. Dev.: 4,2</i>	<i>Mean: 1,10</i> <i>Stand. Dev. 0,5</i>
28 days updating	<i>Mean: 52,6</i> <i>Stand. Dev.: 6,4</i>	<i>Mean: 1,35</i> <i>Stand. Dev.: 0,7</i>
90 days updating	<i>Mean: 51,5</i> <i>Stand. Dev.: 5,2</i>	<i>Mean: 0,95</i> <i>Stand. Dev.: 0,5</i>



Updating of the random variables

- Updating of compressive strength probability density function

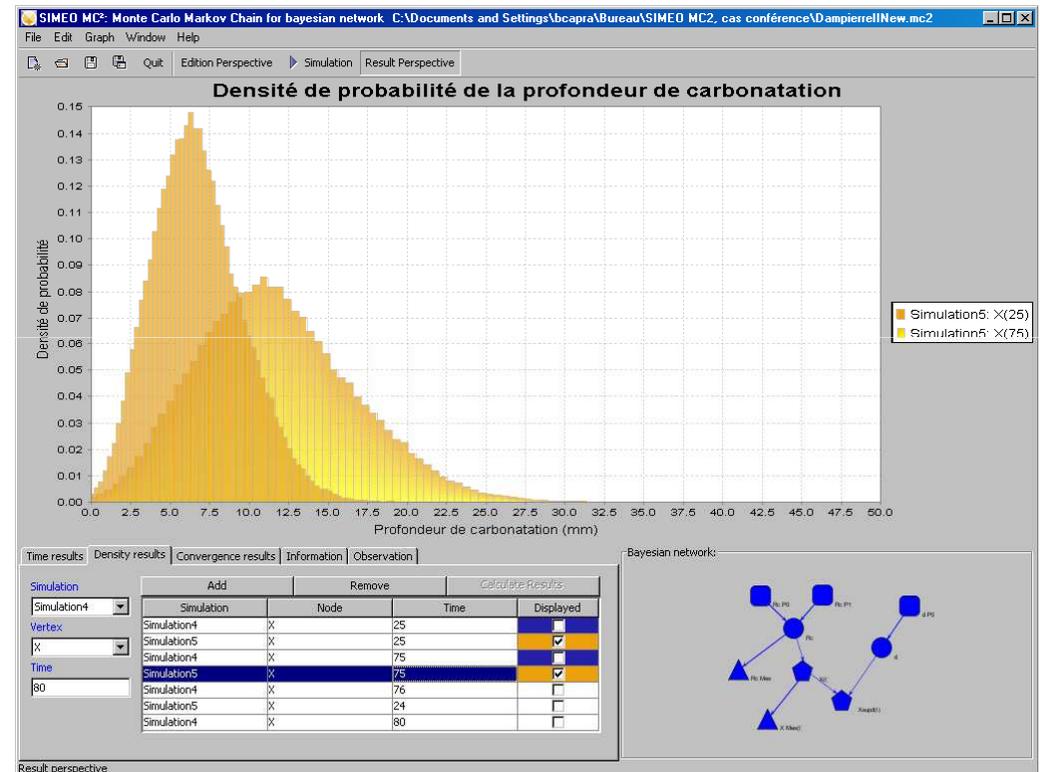
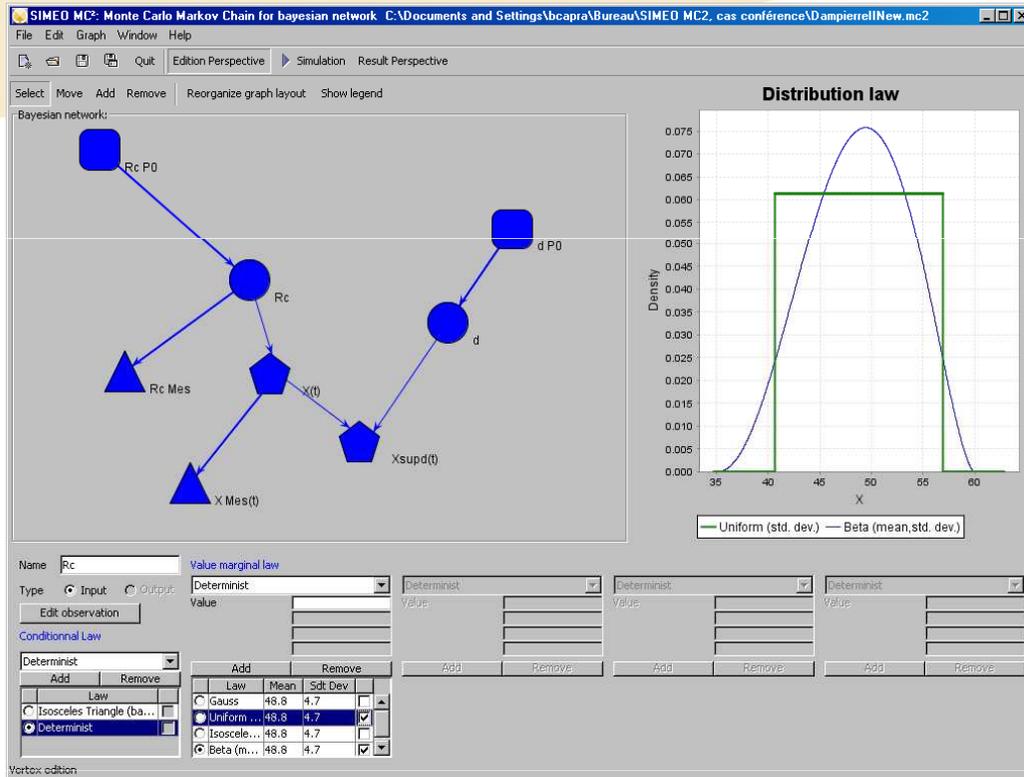




Software: SIMEO MC²

MCMC method (Markov Chains Monte Carlo)

- Markov Chain
- Métropolis-Hastings algorithm with *Gibbs sampling*





CONCLUSIONS

**In the field of infrastructure risk based management,
Bayesian networks allow :**

- the use of field data for updating
(especially where few data are available)
- the use of heterogeneous information
(expert, feedback, tests,...)
- user control of confidence
- a better control of uncertainty for ageing processes

But :

- the method is more complicated than classical reliability approaches (convergence) and must be applied with care
- **does not prevent from non representative models**



OXAND

Optimizing Infrastructure Solutions

Thank you for your attention

OXAND

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