Continuum damage mechanics based model for quasi brittle materials subjected to cyclic loadings: application to concrete

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Context

- Strong dissymmetry between tension and compression

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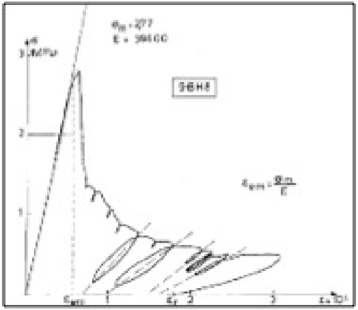
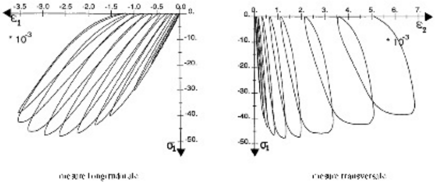
- **Permanent strains in tension**
- **Hysteretic effects**



(TERRIEN, 1980)

Context

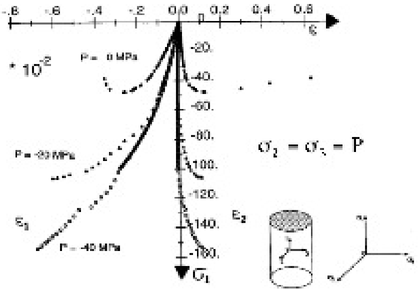
- Strong dissymmetry between tension and compression
- Permanent strains in tension
- Hysteretic effects
- **Permanent strains in compression**
- **Dilatancy**



(RAMTANI, 1990)

Context

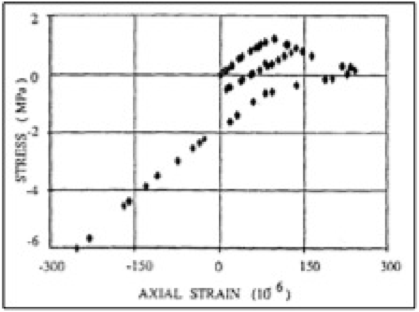
- Strong dissymmetry between tension and compression
- Permanent strains in tension
- Hysteretic effects
- Permanent strains in compression
- Dilatancy
- **Sensitive to the hydrostatic pressure**



(RAMTANI, 1990)

Context

- Strong dissymmetry between tension and compression
- Permanent strains in tension
- Hysteretic effects
- Permanent strains in compression
- Dilatancy
- Sensitive to the hydrostatic pressure
- **Unilateral effect**



(LA BORDERIE, 1991)

Objectives

energie atomique • energies alternatives

The aim of this study is to develop a concrete model such as:

1. the most preponderant dissipative mechanisms are taken into account
2. the cases of cyclic and seismic loadings can be handled

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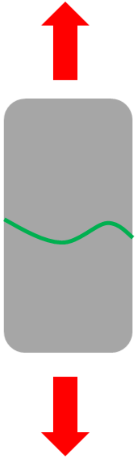
1. Theoretical formulation

Theoretical formulation

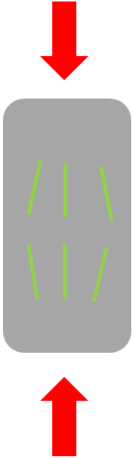
Basic principles

- *In tension*:
  localized cracking



Basic principles

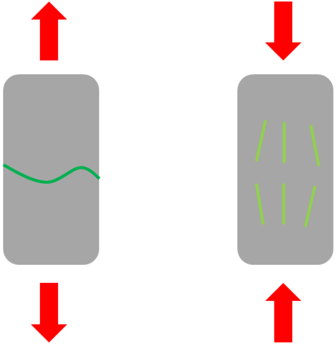
- *In tension*: localized cracking
- *In compression*: diffuse cracking



Theoretical formulation

Assumptions

- *In tension*: damage coupled with hysteretic effects
- *In compression*: plasticity
- Behaviors in tension and compression fully uncoupled



Theoretical formulation

Helmholtz free energy

$$\rho\psi = \frac{1}{2}\epsilon_{ij}C_{ijkl}\epsilon_{kl}$$

Meaning

- Elasticity

Theoretical formulation

Helmholtz free energy

$$\rho\psi = \frac{1}{2}(1-d)\epsilon_{ij}C_{ijkl}\epsilon_{kl}$$

Meaning

- Isotropic damage

Theoretical formulation

Helmholtz free energy

$$\rho\psi = \frac{1}{2}(1-d)\epsilon_{ij}C_{ijkl}\epsilon_{kl} + \frac{1}{2}d(\epsilon_{ij}-\epsilon_{ij}^{\pi})C_{ijkl}(\epsilon_{kl}-\epsilon_{kl}^{\pi})$$

Meaning

- Internal sliding

Theoretical formulation

Helmholtz free energy

$$\rho\psi = \frac{1}{2}(1-d)(\epsilon_{ij} - \epsilon_{ij}^{p})C_{ijkl}(\epsilon_{kl} - \epsilon_{kl}^{p}) + \frac{1}{2}d(\epsilon_{ij} - \epsilon_{ij}^{\pi} - \epsilon_{ij}^{p})C_{ijkl}(\epsilon_{kl} - \epsilon_{kl}^{\pi} - \epsilon_{kl}^{p})$$

Meaning

- Plasticity

Theoretical formulation

Helmholtz free energy

$$\rho\psi = \frac{1}{2}(1-d)(\epsilon_{ij}-\epsilon^p_{ij})C_{ijkl}(\epsilon_{kl}-\epsilon^p_{kl}) + \frac{1}{2}d(\epsilon_{ij}-\eta\,\epsilon^\pi_{ij}-\epsilon^p_{ij})C_{ijkl}(\epsilon_{kl}-\eta\,\epsilon^\pi_{kl}-\epsilon^p_{kl})$$

Meaning

- Unilateral effect

Theoretical formulation

Helmholtz free energy

$$\begin{aligned}\rho\psi &= \frac{1}{2}(1-d)(\epsilon_{ij}-\epsilon^p_{ij})C_{ijkl}(\epsilon_{kl}-\epsilon^p_{kl})\\ &+ \frac{1}{2}d(\epsilon_{ij}-\eta\epsilon^\pi_{ij}-\epsilon^p_{ij})C_{ijkl}(\epsilon_{kl}-\eta\epsilon^\pi_{kl}-\epsilon^p_{kl})\\ &+ H(z)+R(p)+\tfrac{1}{2}\gamma\alpha_{ij}\alpha_{ij}\end{aligned}$$

Meaning

- Hardening

Theoretical formulation

Helmholtz free energy

$$
\begin{aligned}
\rho\psi &= \frac{1}{2}(1-d)(\epsilon_{ij}-\epsilon^{p}_{ij})C_{ijkl}(\epsilon_{kl}-\epsilon^{p}_{kl}) \\
&+ \frac{1}{2}d(\epsilon_{ij}-\eta\epsilon^{\pi}_{ij}-\epsilon^{p}_{ij})C_{ijkl}(\epsilon_{kl}-\eta\epsilon^{\pi}_{kl}-\epsilon^{p}_{kl}) \\
&+ H(z)+R(p)+\frac{1}{2}\gamma\alpha_{ij}\alpha_{ij}
\end{aligned}
$$

Admissibility

- C^2 differentiable (as the sum of regular terms)
- Convex (as the sum of convex terms)

Theoretical formulation

Cauchy stress

$$\sigma_{ij} = \frac{\partial \rho \psi}{\partial \epsilon_{ij}} = C_{ijkl}(\epsilon_{kl} - d\eta\epsilon^{\pi}_{kl} - \epsilon^{p}_{kl})$$

- Continuity with respect to ϵ_{ij}

Theoretical formulation

Cauchy stress

$$\sigma_{ij} = \frac{\partial \rho\psi}{\partial \epsilon_{ij}} = C_{ijkl}(\epsilon_{kl} - d\eta\epsilon^{\pi}_{kl} - \epsilon^{p}_{kl})$$

- Continuity with respect to ϵ_{ij}

Sliding stress

$$\sigma^{\pi}_{ij} = -\frac{\partial \rho\psi}{\partial \epsilon^{\pi}_{ij}} = dC_{ijkl}(\epsilon_{kl} - \epsilon^{\pi}_{kl} - \epsilon^{p}_{kl})$$

- Frictional stress between the lips of the cracks

Theoretical formulation

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Damage threshold surface

$$f_d = \bar{Y} - (Y_0 + Z) \leq 0$$

$$\bar{Y} = \frac{1}{2} < \epsilon_{ij} - \epsilon_{ij}^{p} >_{+} C_{ijkl} < \epsilon_{kl} - \epsilon_{kl}^{p} >_{+}$$

Theoretical formulation

Damage threshold surface

$$f_d = \bar{Y} - (Y_0 + Z) \leq 0$$

$$\bar{Y} = \frac{1}{2} < \epsilon_{ij} - \epsilon_{ij}^p >_+ C_{ijkl} < \epsilon_{kl} - \epsilon_{kl}^p >_+$$

Flow rules

$$\dot{d} = \dot{\lambda}_d \frac{\partial f_d}{\partial \bar{Y}} = \dot{\lambda}_d$$

$$\dot{z} = \dot{\lambda}_d \frac{\partial f_d}{\partial Z} = -\dot{\lambda}_d$$

$$H(z) = \frac{-z}{A_d(1-z)}$$

Theoretical formulation

Damage threshold surface

$$f_d = \bar{Y} - (Y_0 + Z) \leq 0$$

$$\bar{Y} = \frac{1}{2} < \epsilon_{ij} - \epsilon_{ij}^p >_+ C_{ijkl} < \epsilon_{kl} - \epsilon_{kl}^p >_+$$

Flow rules

- Explicit integration of the damage
- Numerical robustness

$$d = 1 - \frac{1}{1 + A_d(\bar{Y} - Y_0)}$$

Theoretical formulation

Internal sliding threshold surface

$$f_\pi = J_2(\sigma^\pi - X) \leq 0$$

$$\phi_\pi = J_2(\sigma^\pi - X) + \frac{a}{2} X_{ij} X_{ij}$$

Theoretical formulation

Internal sliding threshold surface

$$f_\pi = J_2(\sigma^\pi - X) \leq 0$$

$$\phi_\pi = J_2(\sigma^\pi - X) + \frac{a}{2} X_{ij} X_{ij}$$

Flow rules

$$\dot{\epsilon}_{ij}^\pi = \dot{\lambda}_\pi \frac{\partial f_\pi}{\partial \sigma_{ij}^\pi}$$

$$\dot{\alpha}_{ij} = -\dot{\lambda}_\pi \frac{\partial \phi_\pi}{\partial X_{ij}}$$

Theoretical formulation

Internal sliding threshold surface

$$f_\pi = J_2(\sigma^\pi - X) \leq 0$$

$$\phi_\pi = J_2(\sigma^\pi - X) + \frac{a}{2} X_{ij} X_{ij}$$

Flow rules

$$\dot{\epsilon}^\pi_{ij} = \dot{\lambda}_\pi \frac{\partial f_\pi}{\partial \sigma^\pi_{ij}}$$

$$\dot{\alpha}_{ij} = -\dot{\lambda}_\pi \frac{\partial \phi_\pi}{\partial X_{ij}}$$

- Implicit integration scheme
- Quadratic convergence

Theoretical formulation

energie atomique • energies alternatives

Plasticity threshold surface

$$f_p = J_2(\sigma) + \alpha_f I_1(\sigma) - (f_0 + R) \leq 0$$

$$\phi_p = J_2(\sigma) + \alpha_\phi I_1(\sigma)$$

Theoretical formulation

Plasticity threshold surface

$$f_p = J_2(\sigma) + \alpha_f I_1(\sigma) - (f_0 + R) \leq 0$$
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Flow rules

$$\dot{\epsilon}_{ij}^p = \dot{\lambda}_p \frac{\partial \phi_p}{\partial \sigma_{ij}}$$
$$\dot{p} = -\dot{\lambda}_p \frac{\partial f_p}{\partial R} = \dot{\lambda}_p$$

Theoretical formulation

Plasticity threshold surface

$$f_p = J_2(\sigma) + \alpha_f I_1(\sigma) - (f_0 + R) \leq 0$$

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Flow rules

$$\dot{\epsilon}^p_{ij} = \dot{\lambda}_p \frac{\partial \phi_p}{\partial \sigma_{ij}}$$

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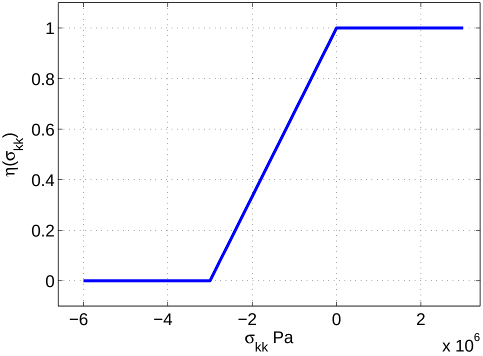
- Implicit integration scheme
- Quadratic convergence

Unilateral conditions

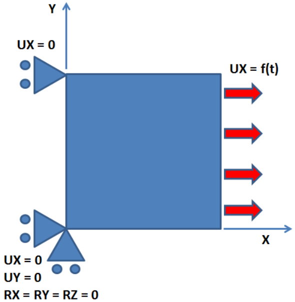


(LA BORDERIE ET AL, 1991)

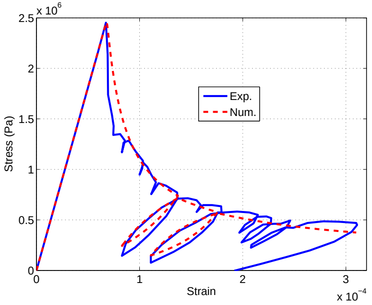
2. Elementary tests

Elementary tests

Cyclic tension test



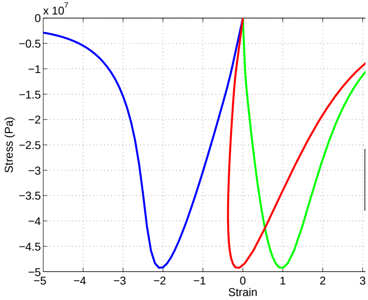
Numerical results



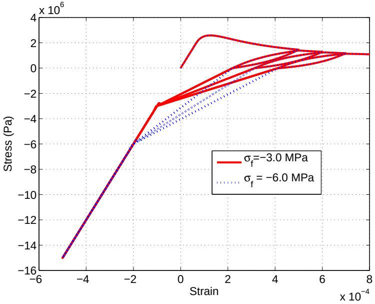
(TERRIEN, 1980)

Elementary tests

Compression

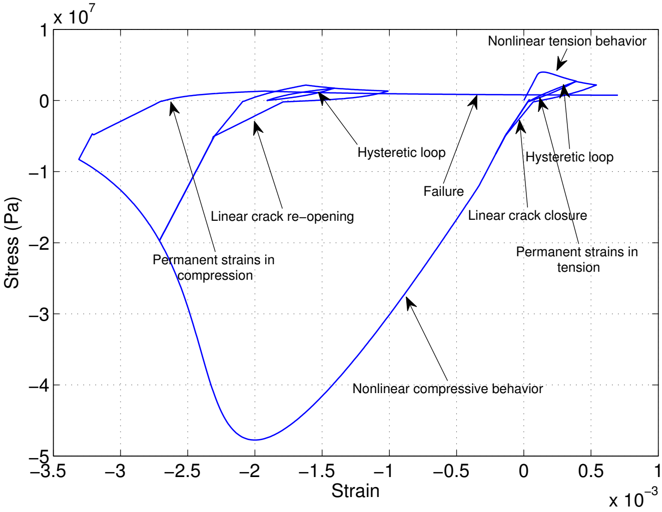


Unilateral effect



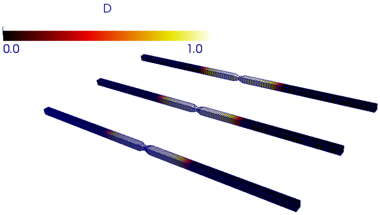
Elementary tests

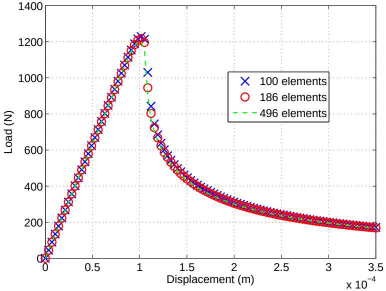
Global stress/strain response



Elementary tests

Regularization: nonlocal (Pijaudier-Cabot et al, 1987)





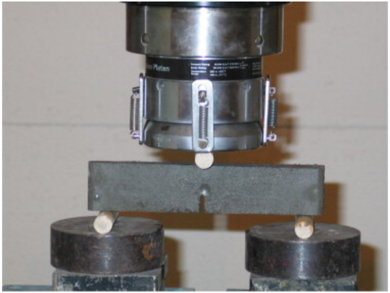
- Damage pattern mesh-independent
- Dissipated energy mesh-independent

3. Structural examples

Structural examples

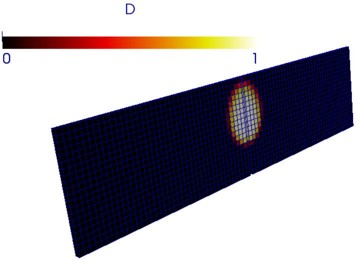
Single edge notched concrete beam

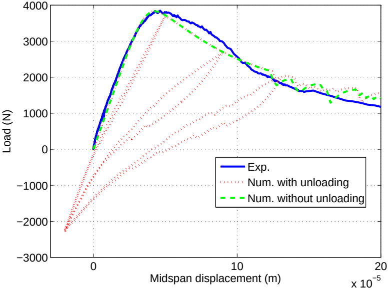
- Three-point-bending test
- Stability of the crack propagation
- **Capability of the model to open and to close a single crack**



Structural examples

Single edge notched concrete beam





- Nonlinear hysteretic effect coupled with damage
- Meaningful monotonic response
- Pinch effect taken into account

Structural examples

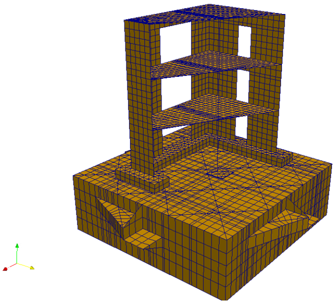
RC mock-up subjected to seismic loadings

- REV of an electrical building of a nuclear power plant
- 3D nonlinear effects coupling torsion and shear
- 2D Seismic loading with a PGA equal to 1.0g

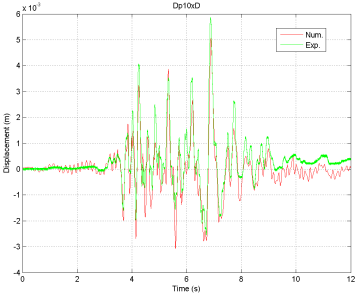
RC mock-up subjected to seismic loadings

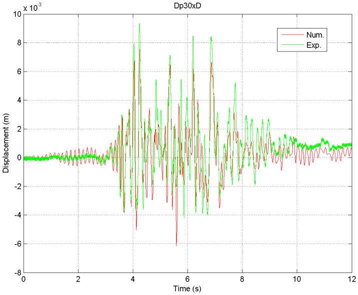
- **Concrete**: proposed model
- **Steel**: Menegotto-Pinto's model (MENEGOTTO ET AL, 1973)
- **Shaking table**: elastic

(CAST3M-CEA, 2005)



RC mock-up subjected to seismic loadings

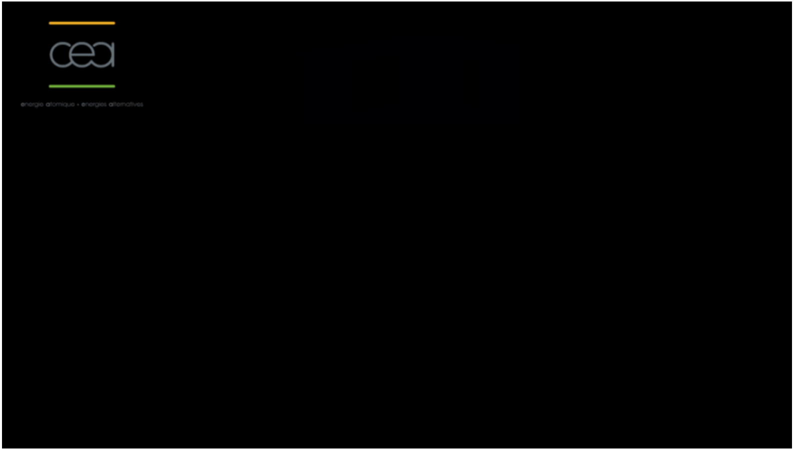




- Accurate description of the time history displacement
- Satisfactory results

Structural examples

RC mock-up subjected to seismic loadings



4. Concluding remarks and outlooks

Structural examples

Concluding remarks

- Continuum damage mechanics based model for seismic applications
- Implementation in Cast3M-CEA
- First results are satisfactory

Outlooks

- Capability to reduce the viscous damping matrix (2% ?, 0.8%, etc...)
- Identification of the unilateral conditions (nonlinear crack closure ?, criterion ?, etc...)

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