

# Cracking of Concrete Structures Interest and Advantages of Probabilistic Approaches

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# Objectives

- Presentation of the **strategy of modelling** of the cracking of concrete structures developed at IFSTTAR: **probabilistic models**
- Presentation of some examples of applications

# Some consideration about the physical mechanisms which fund the cracking of concrete structures



- Cracks are created and propagate due to the presence of tensile stresses
  - Cracks creation is dependent of the concrete heterogeneity



The concrete tensile strength is volume effect dependent: **theory of the weakness link**

- Until the tensile peak stress, concrete is quasi-elastic: **creation of microcracks diffused in all the stressed volume of material**  
Material behaviour
- Concrete post-peak behaviour in tension is the consequence of crack localization: **macrocrack propagation**  
Structural behaviour

# Current deterministic modellings



- **Discrete cracking models: linear and non-linear fracture mechanics**
- **Distributed damage models: smeared and damage models**

**The cracking energy** is independent of the **scale of modelling**, that means independent of the **mesh elements size** (finite element analysis)

# Discrete cracking models

**Analysis of one macrocrack propagation  
concrete dams for example**

**Not well adapted for cracking analysis of  
reinforced concrete structures**

# Distributed damage models

Well adapted for analysis of multi-macrocracking of concrete structures

**Difficulty:** getting precise information about **cracks opening**



$\epsilon$    $W$

Characteristic length ?

# Probabilistic modellings

- **First development by Rossi et al (1985)**
- **First paper, 1987 – *Material and Structures***

## 3 mean assumptions

- The concrete tensile strength is a **probabilistic** parameter:  
**Material heterogeneity**
- The concrete tensile strength is a **scale effect** parameter  
which depends on  
**Stressed Volume of Material/Maximum Aggregate Volume**
- A volume of **mesh element** (finite elements computations)  
is equivalent to the same **volume of concrete**

**Deterministic computation with probabilistic variables**



**N computations**

**Different spatial distribution of the variables**



**Monte Carlo approach**



**Safety Analysis of the structures**

# Strategy

## Different modelling scales



- **Local scale** (focus on a locale zone of the whole structure or structural element)
- **Global scale** (the whole structure or structural element)

## Strategy

# Local scale

The scale of modelling (finite elements size) is **very small** compared to the **structure dimensions** and the **stresses gradients**



**Perfect brittle behaviour** of the material  
**Probabilistic distribution** of the **tensile strength**

## Strategy

# Local scale

**Numerical support: reasonable computation time**



- 2D computations: use of **interface elements**



**Discrete approach: Creation of real discontinuities of displacement (cracks creations)**

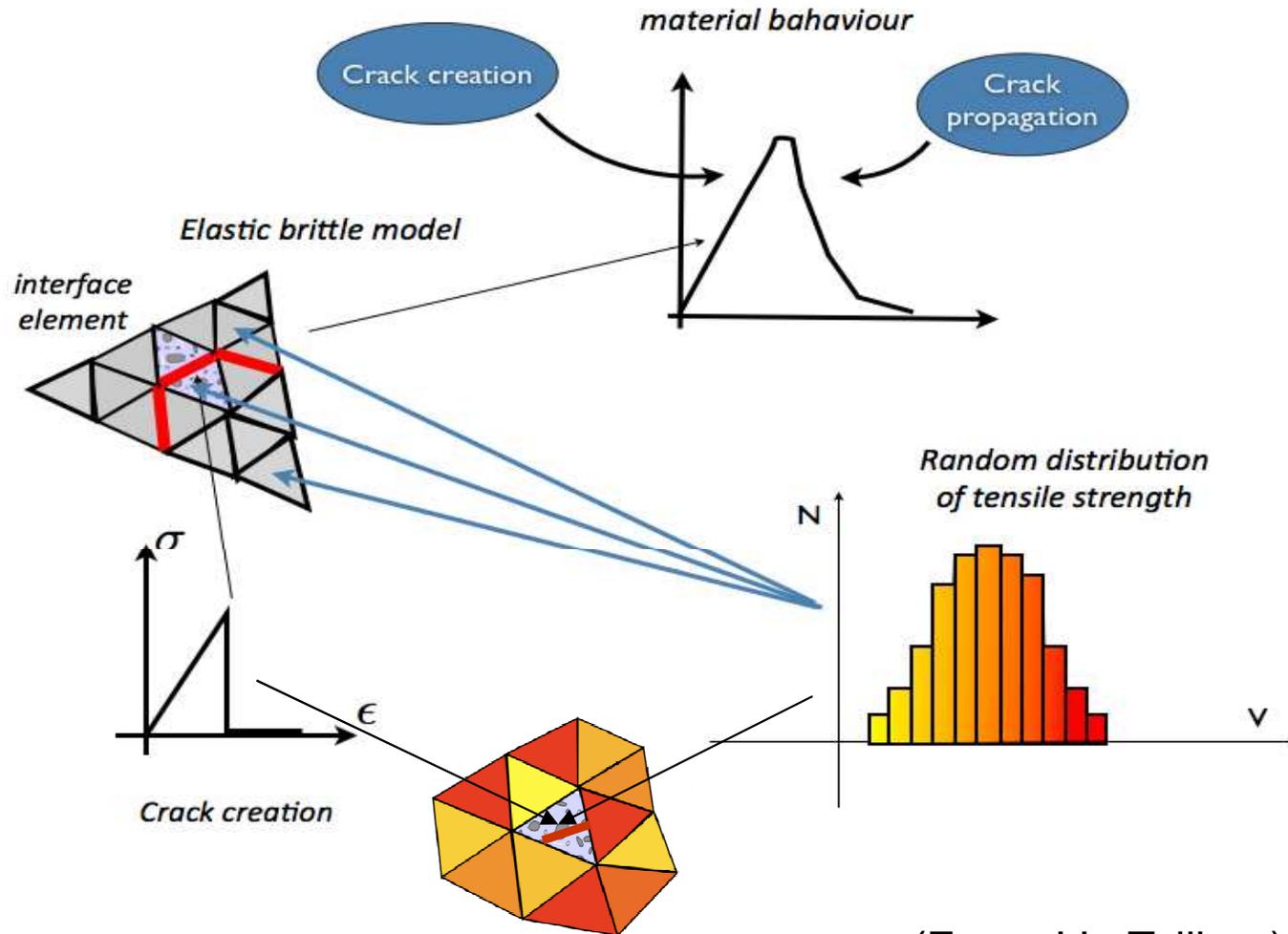
- 3D computations: use of **volume elements**



**Macroscopic approach: Creation of holes**

# Strategy

## Local scale



(From J.L. Tailhan)

## Strategy

### Global scale

The scale of modelling includes cracks **creation** and **propagation**



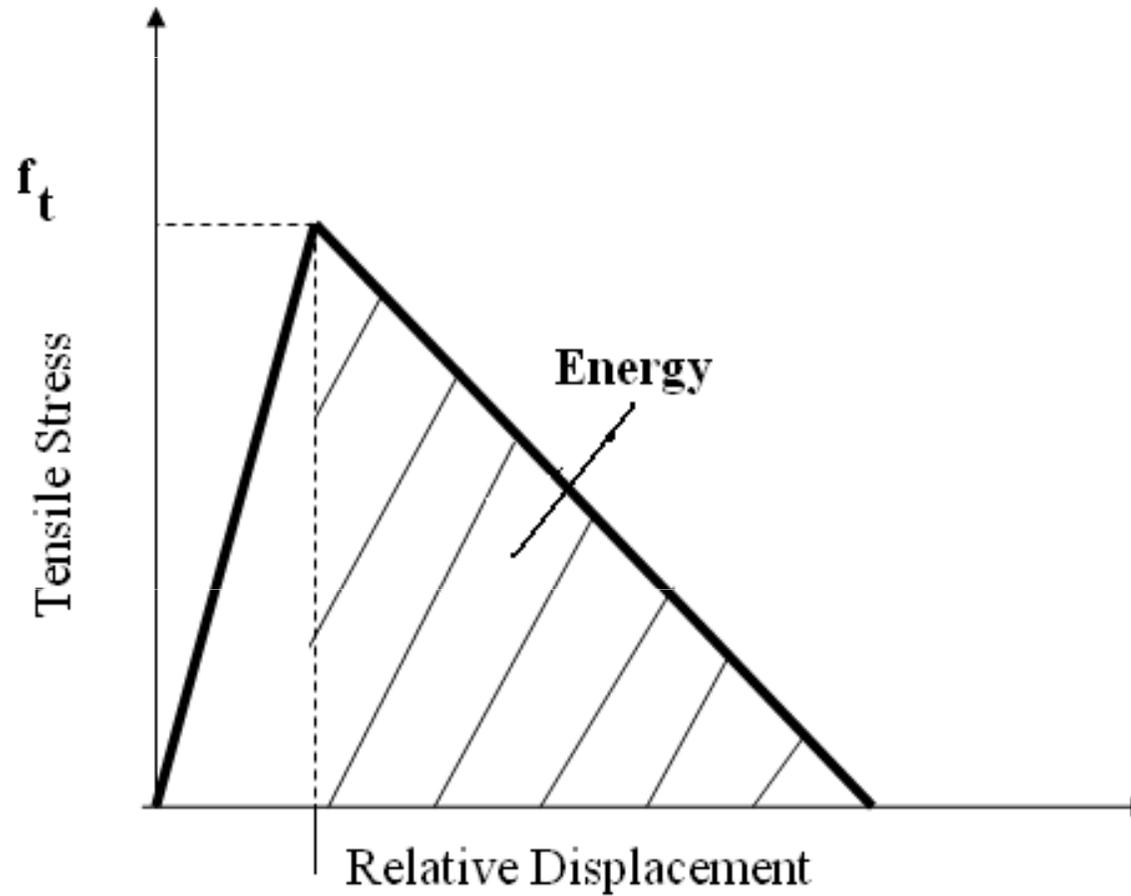
- **Cracks creation (initiation):** Probabilistic distribution of the **tensile strength**
  - Cracks propagation: **damage model**



The tensile stress **lineary decreases** with the **relative displacement** (same direction that the one related to the stress) in the finite elements: Probabilistic distribution of the **post-cracking energy**

# Strategy

## Global scale



## Strategy

# Global scale

**Same strategy than for local scale concerning the numerical support adopted**



- **2D Computations: interface elements**
- **3D Computations: volume elements**

# Examples of application

1. **Macrocrack propagation in a Double Cantilever Beam**  
**(Linear Fracture Mechanics, Rossi, 1986)**



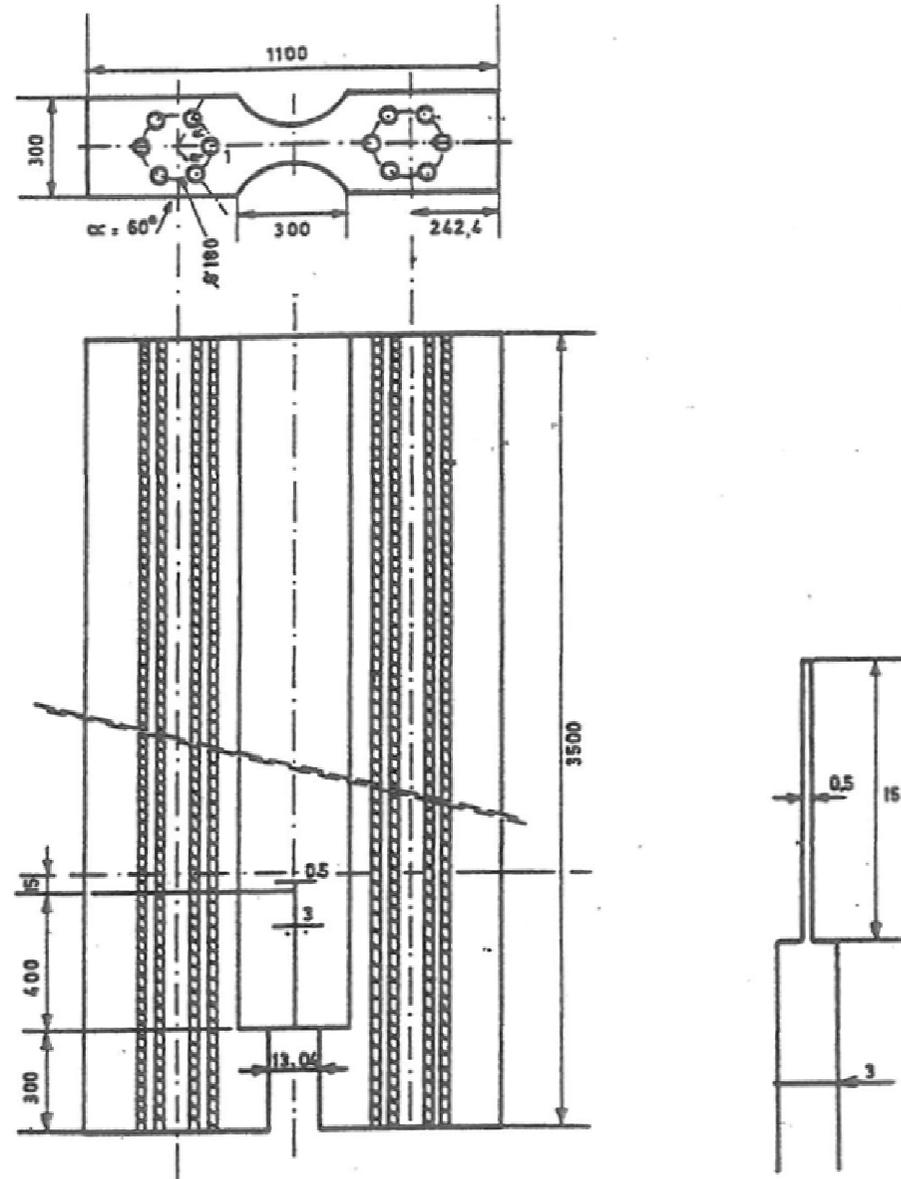
**2D Analysis: local and global approaches**  
**Tailhan et al. Session I-4, 11h-12h20**

2. **Reinforced Concrete Tie-Beam**  
**(Phan et al, 2012)**



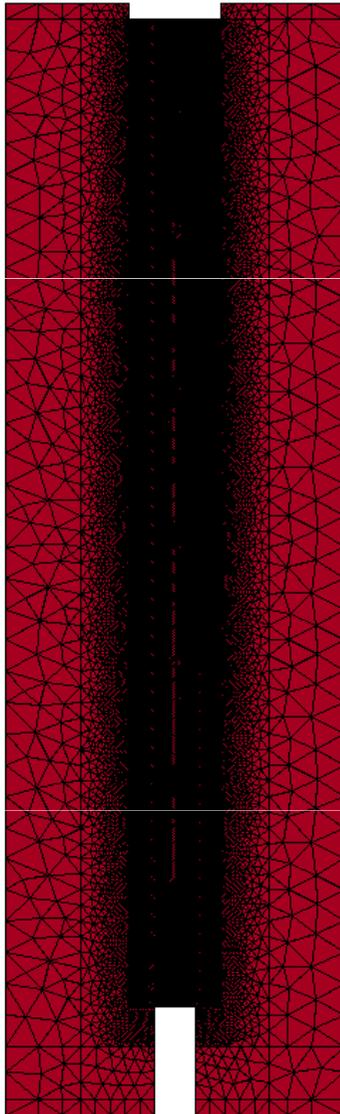
**3D Analysis: local approach**  
**Phan et al. Session I-4, 2h-3h20**

## Macrocrack propagation in a Double Cantilever Beam

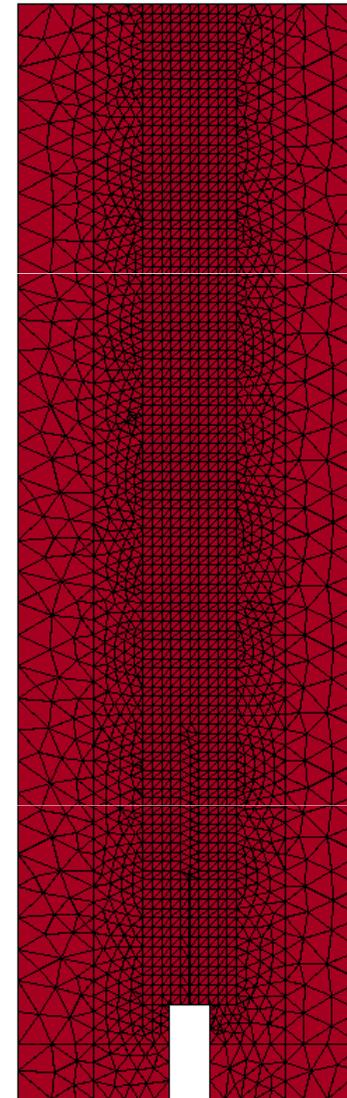


UNIT : mm

## Macrocrack propagation in a Double Cantilever Beam

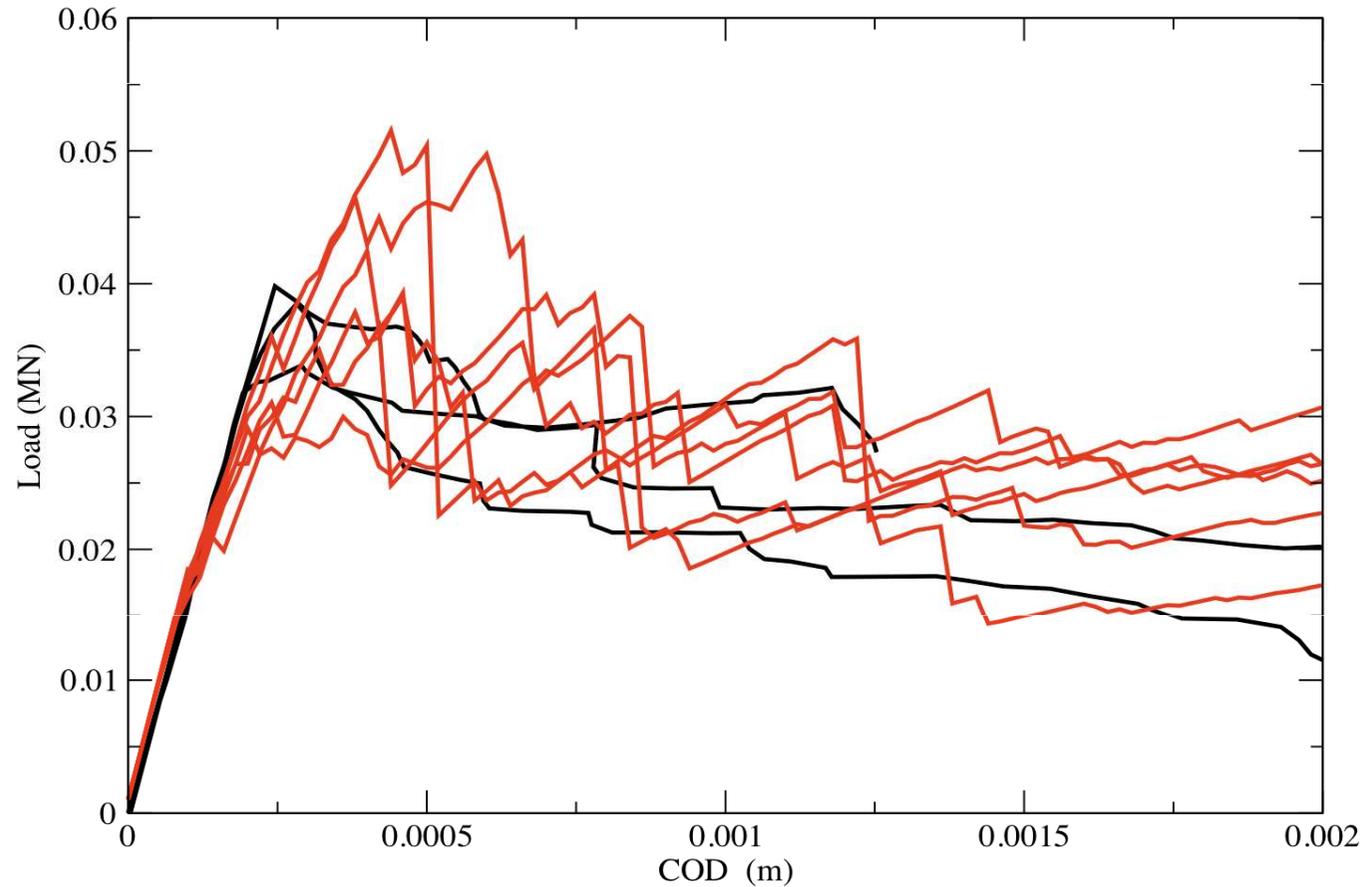


Finite element mesh: local model



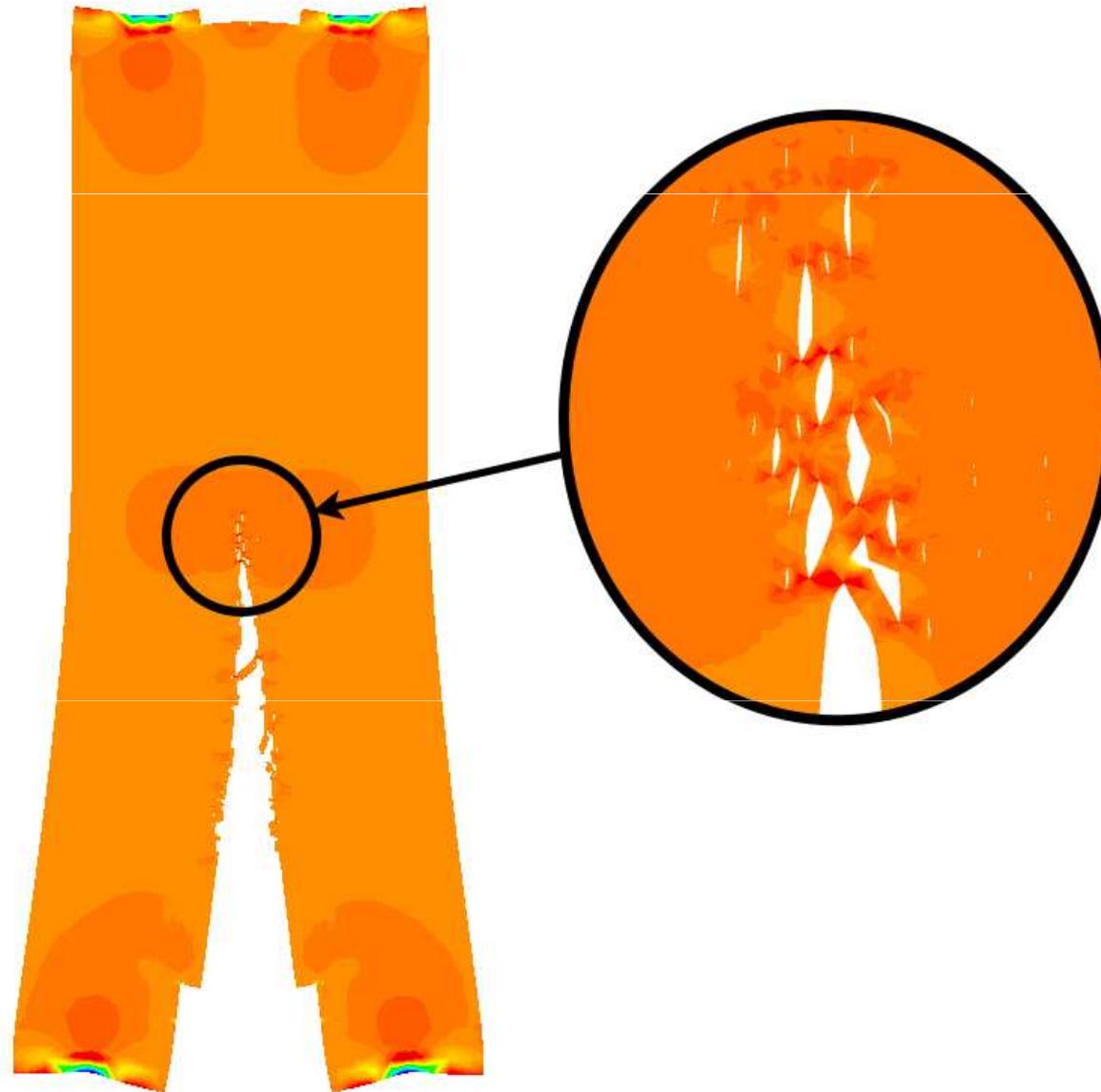
Finite element mesh: global approach

# Macrocrack propagation in a Double Cantilever Beam: Local approach

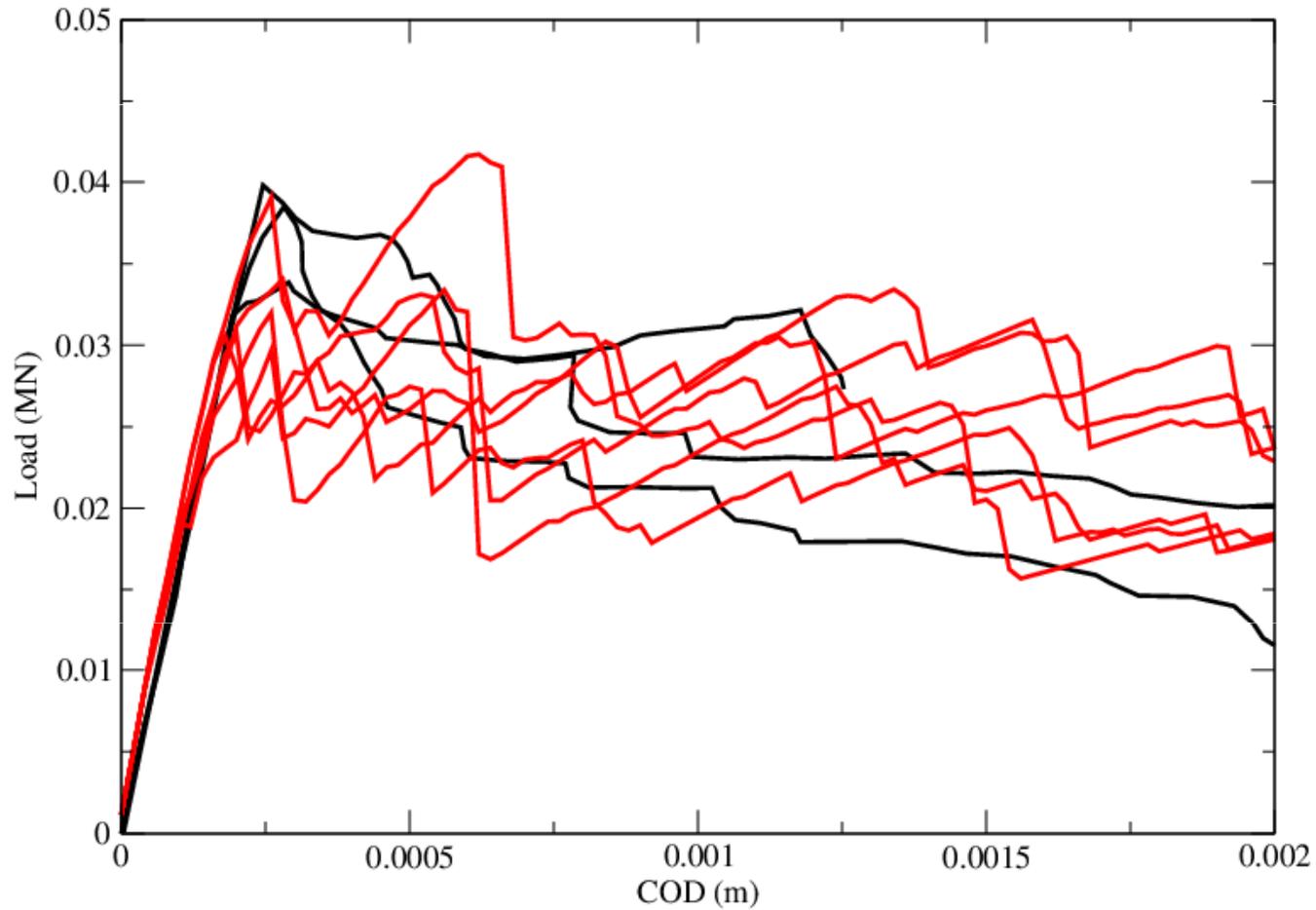


**Red: Computations**  
**Black: Experience**

## Macrocrack propagation in a Double Cantilever Beam: Local approach

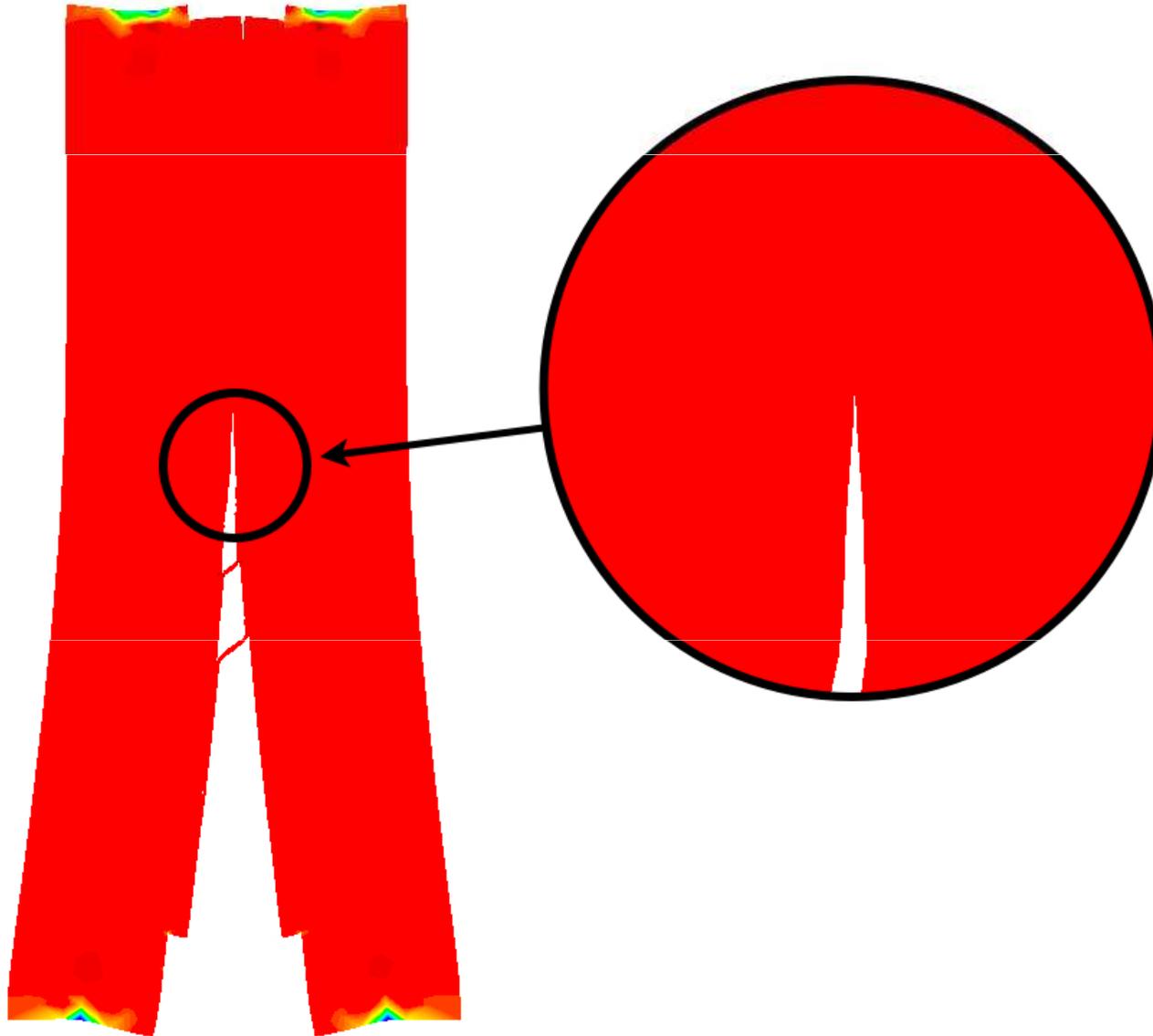


# Macrocrack propagation in a Double Cantilever Beam: **Global approach**

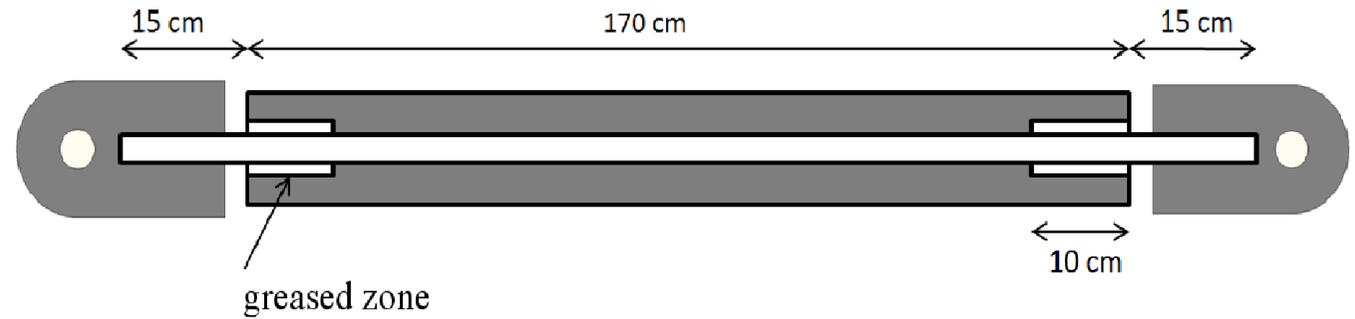


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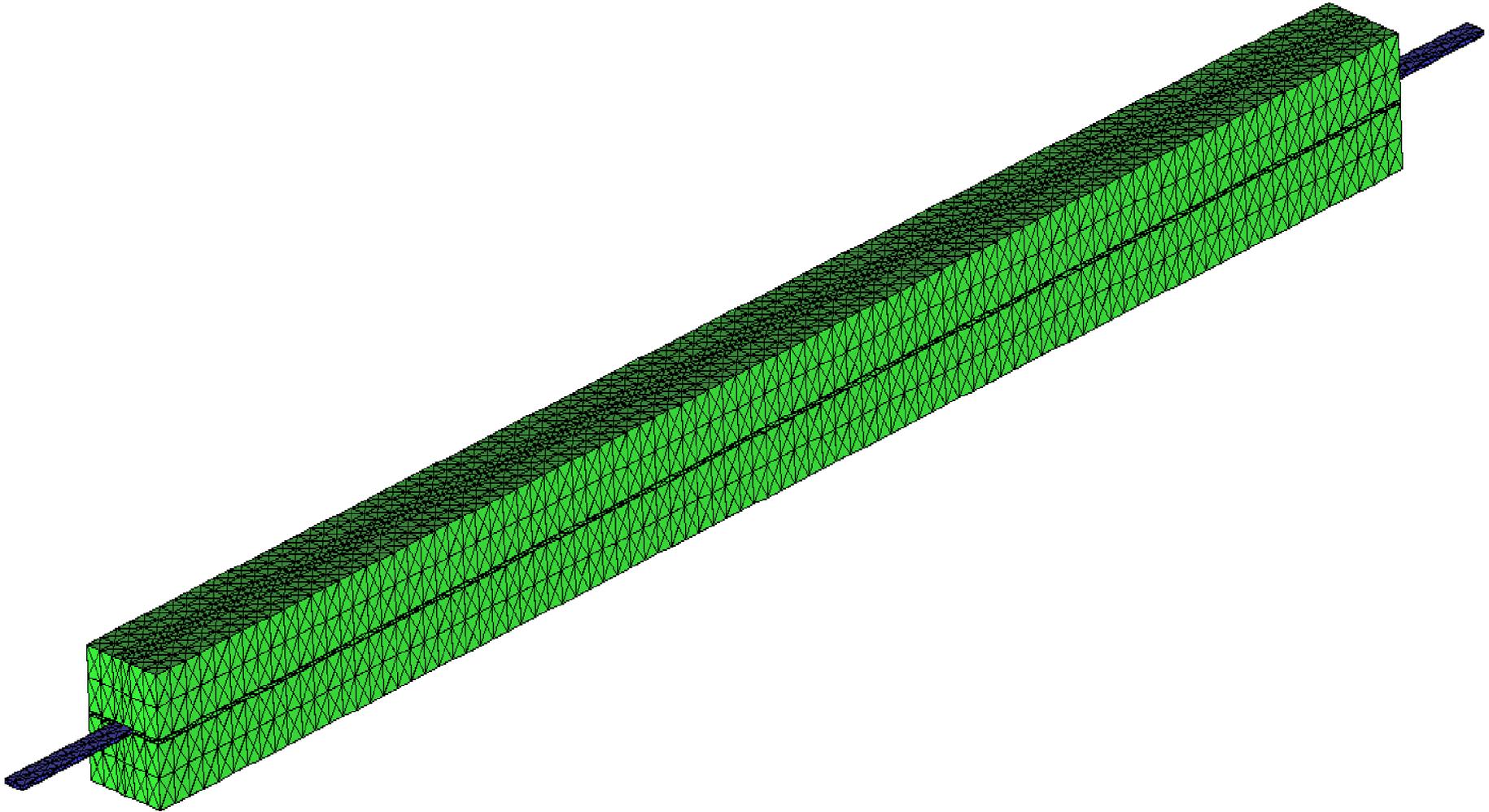
## Macrocrack propagation in a Double Cantilever Beam: **Global approach**



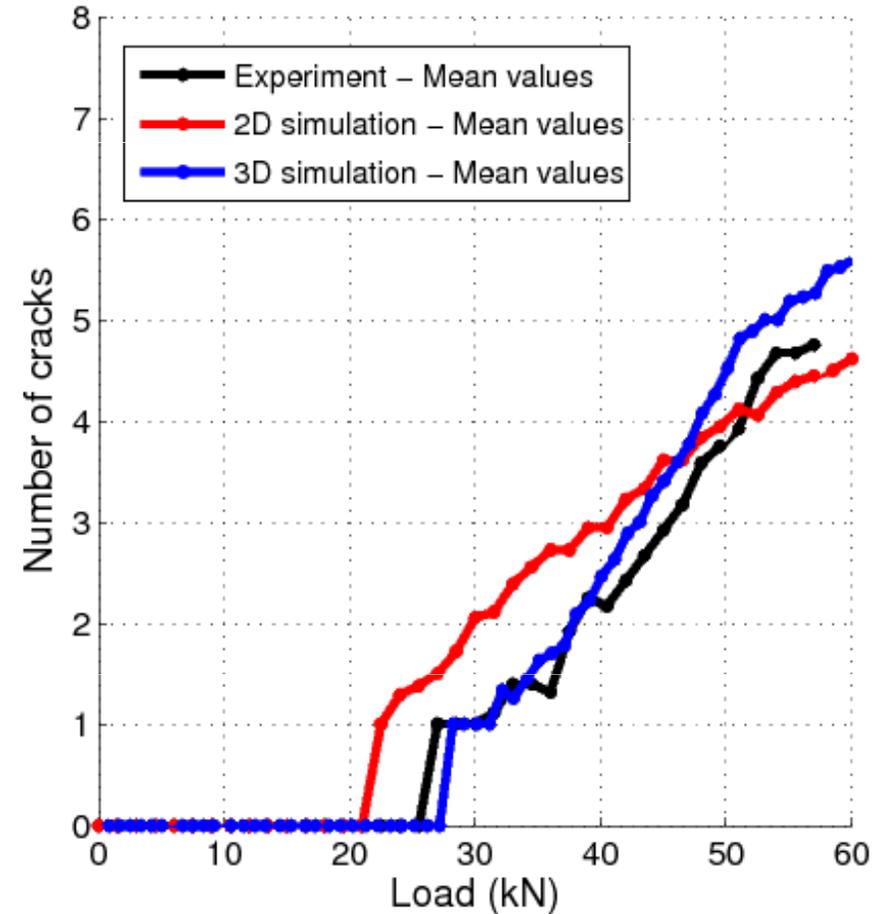
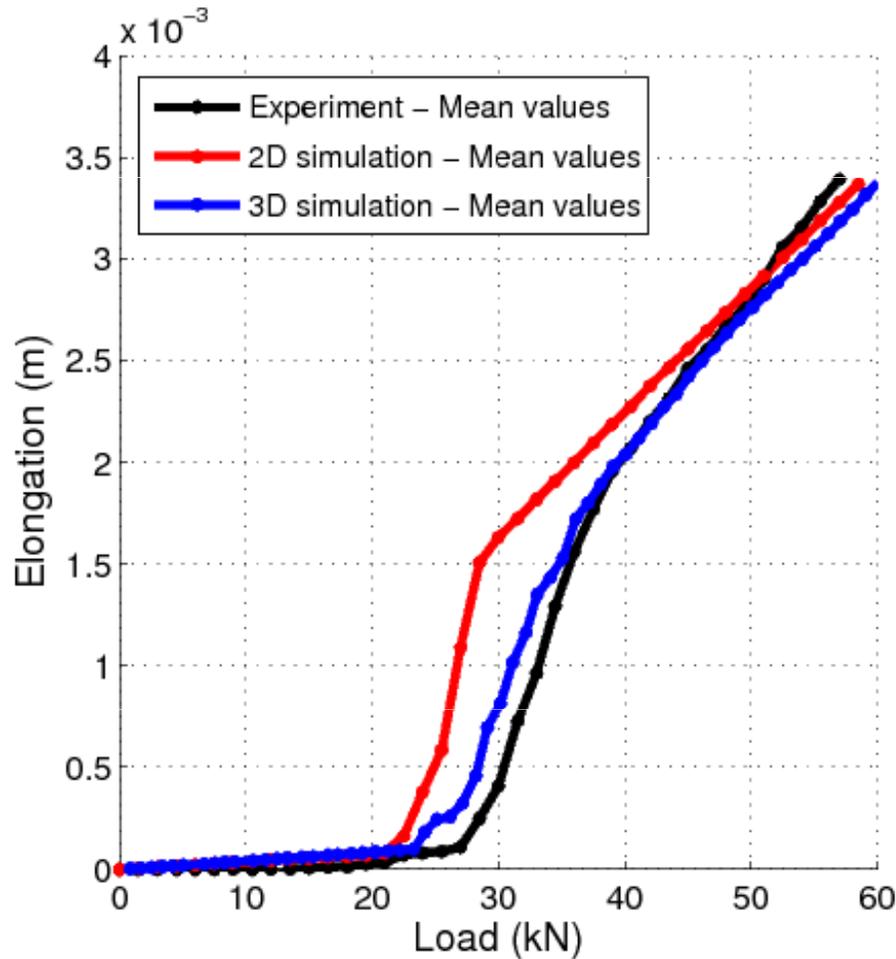
## Reinforced Concrete Tie-Beam: **Global approach**



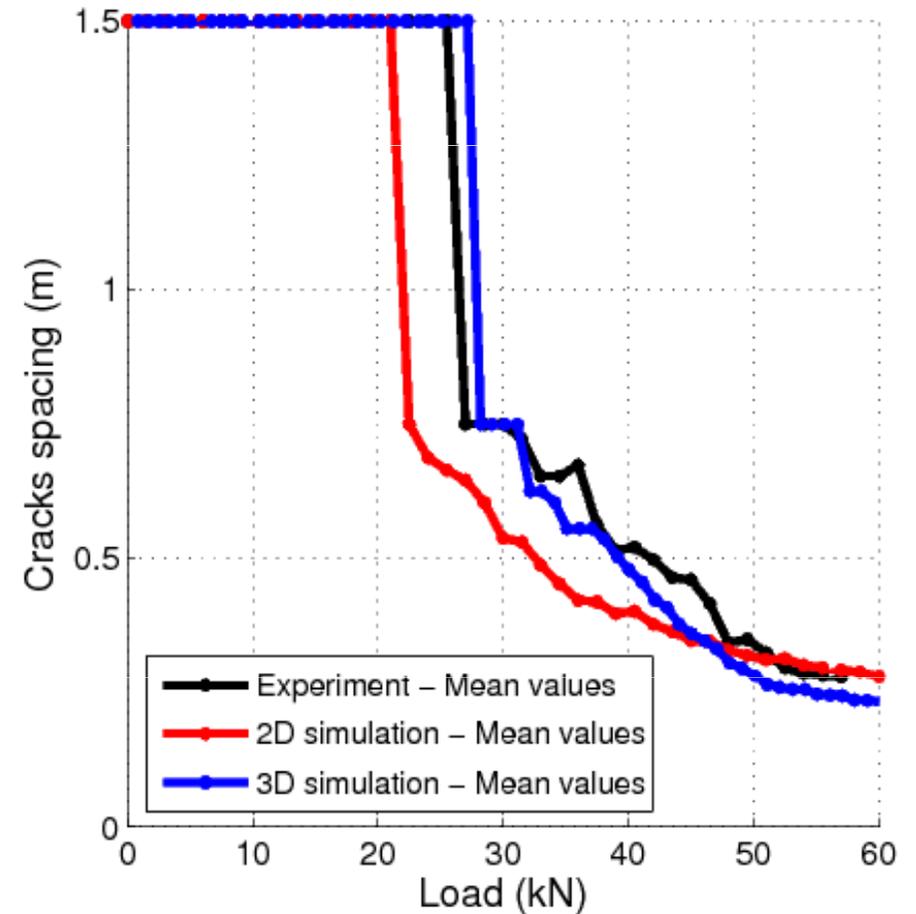
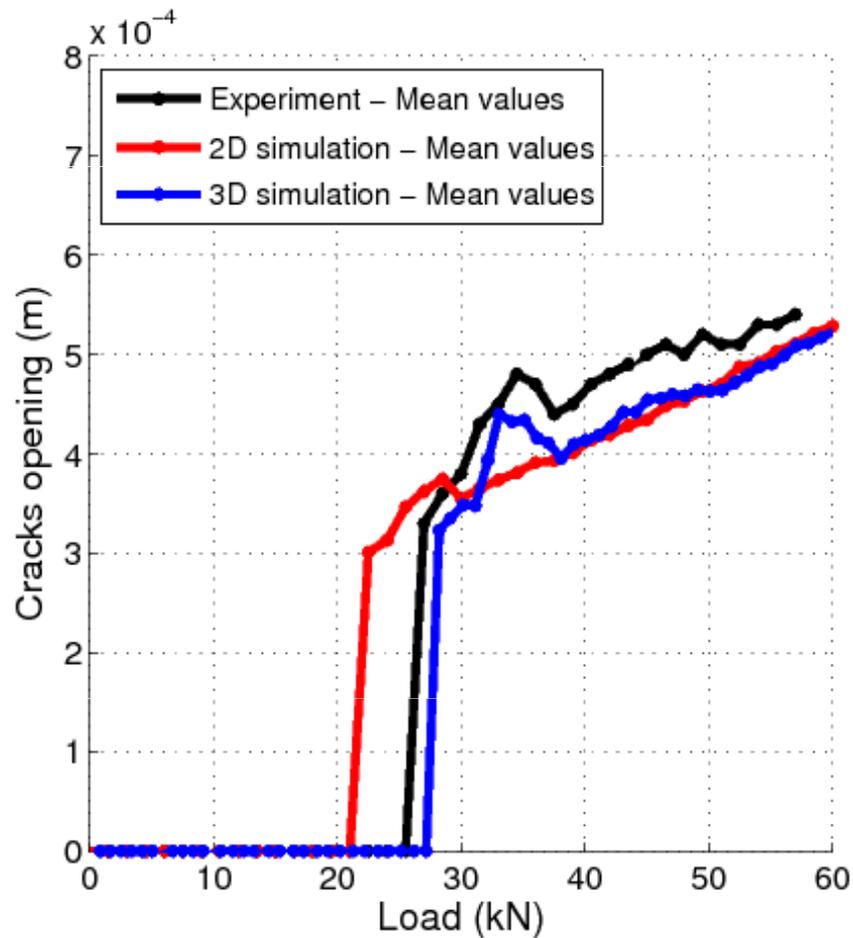
## Reinforced Concrete Tie-Beam: **Global approach**



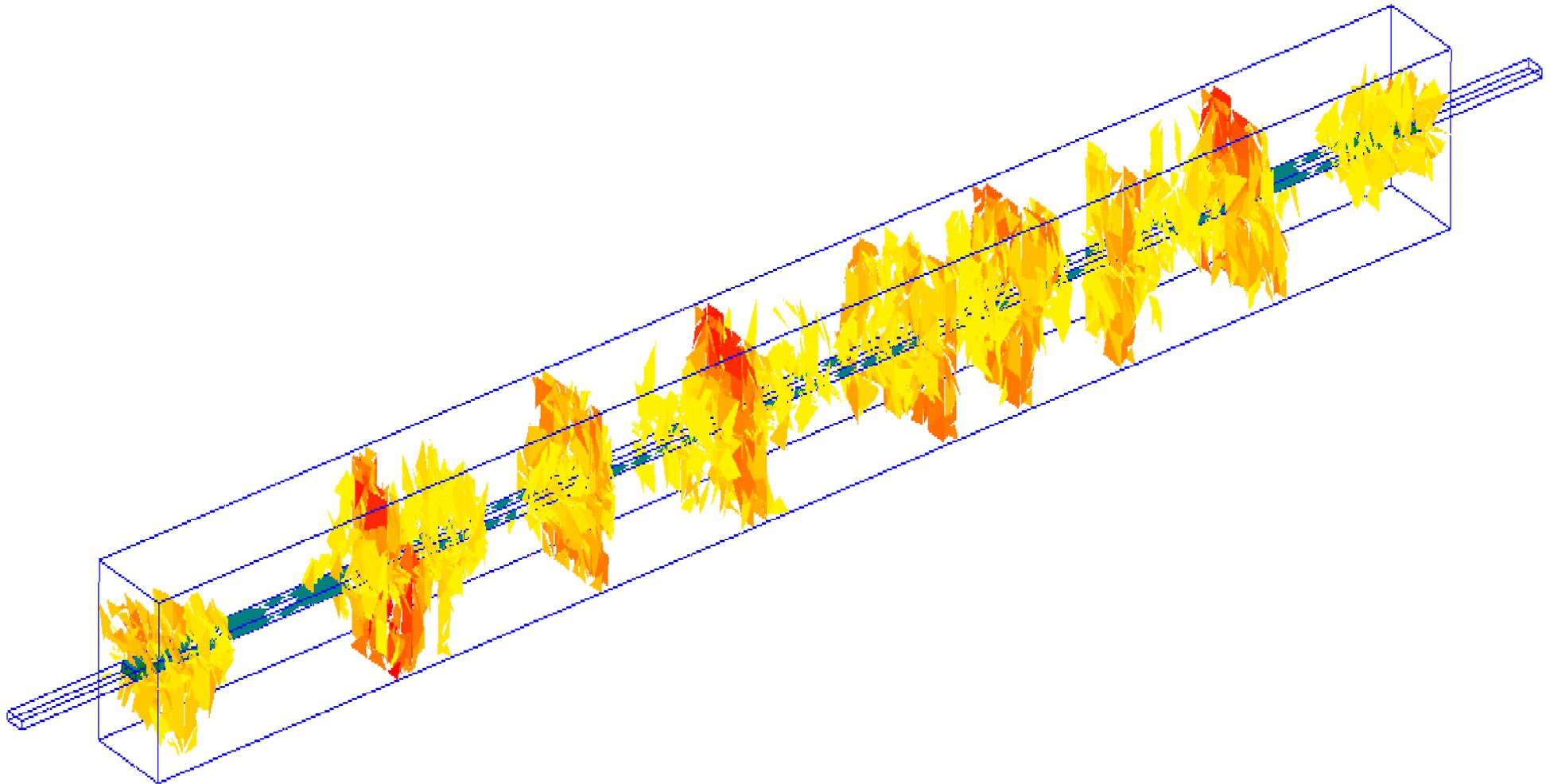
# Reinforced Concrete Tie-Beam: Global approach



# Reinforced Concrete Tie-Beam: Global approach



# Reinforced Concrete Tie-Beam: **Global approach**



# CONCLUSIONS

- Developpement of a **modelling strategy** for analysing cracking of concrete structures at different scales
  - These modellings are **probabilistic ones: deterministic computations with probabilistic variables**  $\Rightarrow$  N Computations in the frame of **Monte Carlo** procedure (**Safety analysis**)
  - The modellings are **physically based** but very **simple** with **few materials variables**
- The modellings give **global (global mechanical behaviour)** and **local (cracks opening, spacing...)** informations
  - **The first results are very promising**