Numerical Modelling
Strategies for Sustainable Concrete Structures

Probabilistic modelling of crack creation and propagation in concrete structures: some numerical and mechanical considerations.

J.-L. Tailhan, P. Rossi, T.S. Phan, J. Foulliaron

IFSTTAR

French Institute of Sciences and Technology for Transport, Development and Networks, Paris, France.

LAYOUT

CRACKING PROCESSES IN CONCRETE

PROBABILISTIC DISCRETE CRACKING MODEL

EXAMPLES AND LIMITS OF THE MODEL

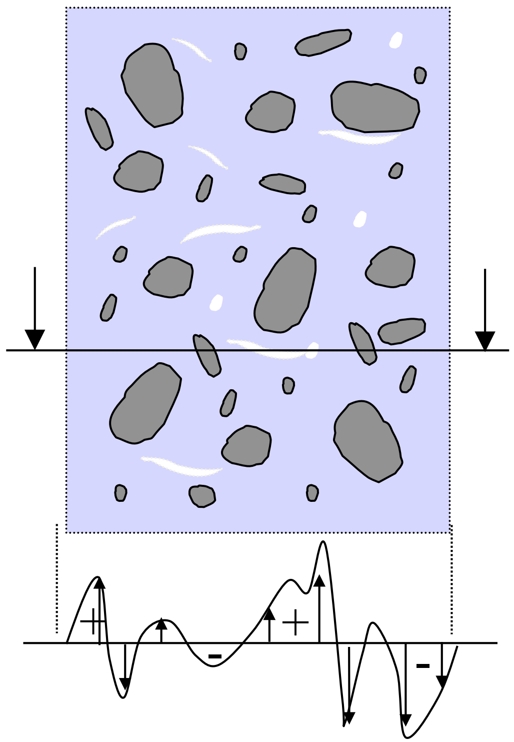
PROBABILISTIC COHESIVE CRACKING MODEL

VALIDATION EXAMPLE

CONCLUSIONS

CRACKING PROCESSES IN CONCRETE

Heterogénéity
& initial defects
ramdomly distributed

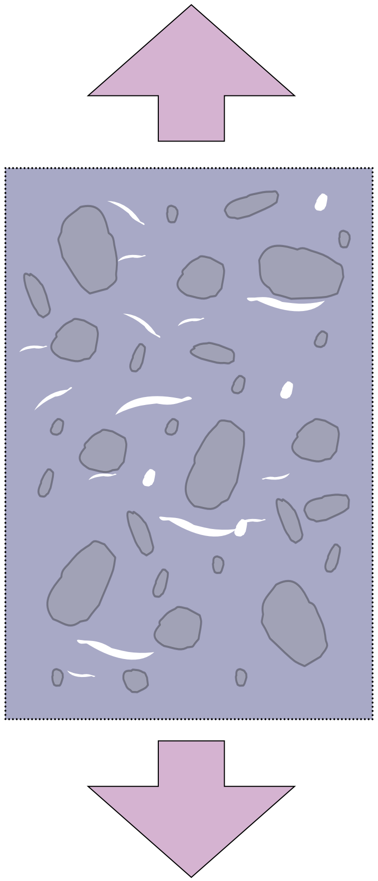


Concrete is a heterogeneous material
Different components: Cement paste + agregates

Initial defects
Pores, cracks

Auto-balanced stresses
Early age restrained shrinkages

CRACKING PROCESSES IN CONCRETE



APPLIED LOAD

+

Heterogeneity => internal stress gradients

Initial defects => starting points of cracks

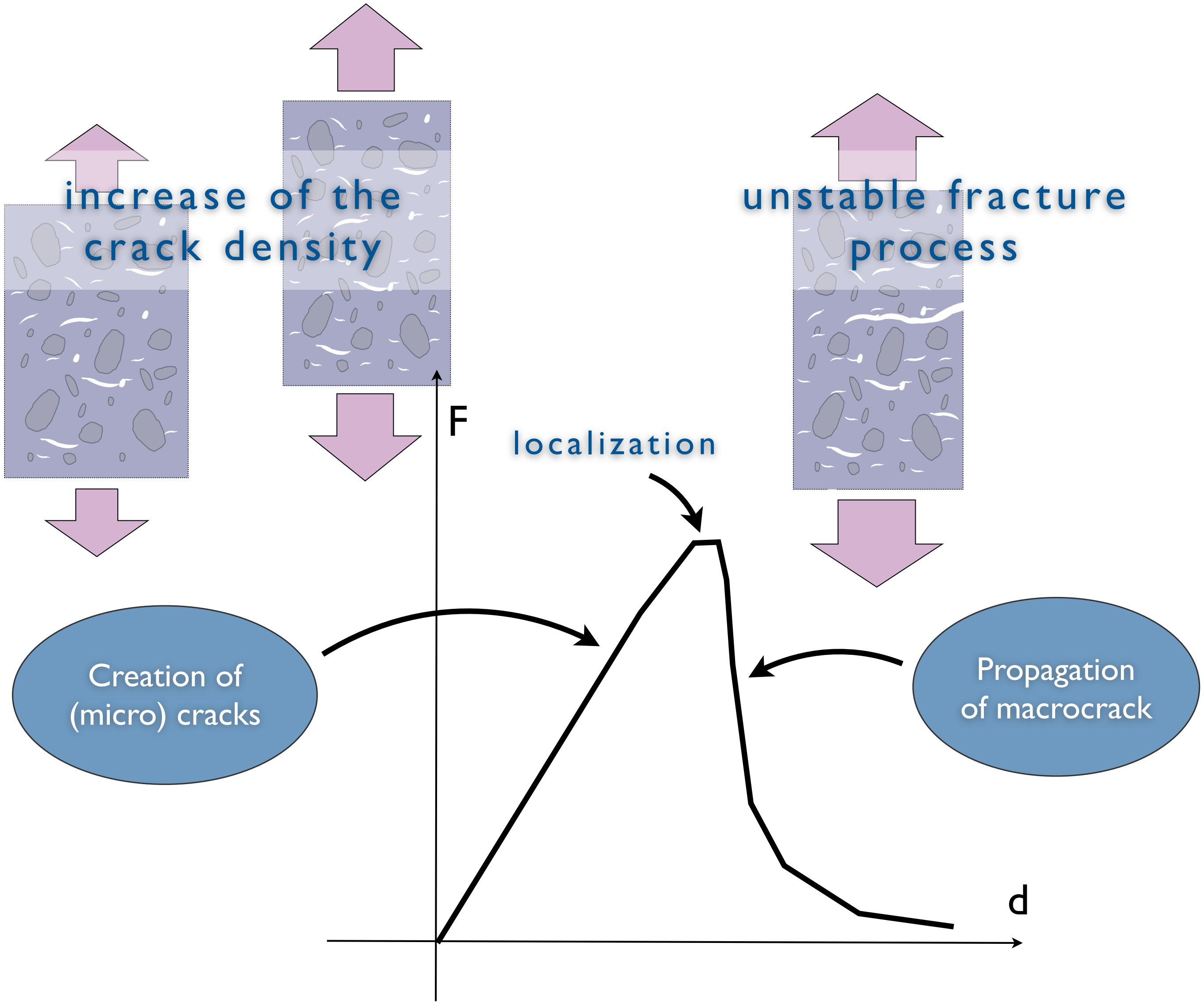
=

Heterogénéity & initial defects ramdomly distributed

↓

Cracking processes at the material scale

CRACKING PROCESSES IN CONCRETE



Heterogénéity & initial defects ramdomly distributed

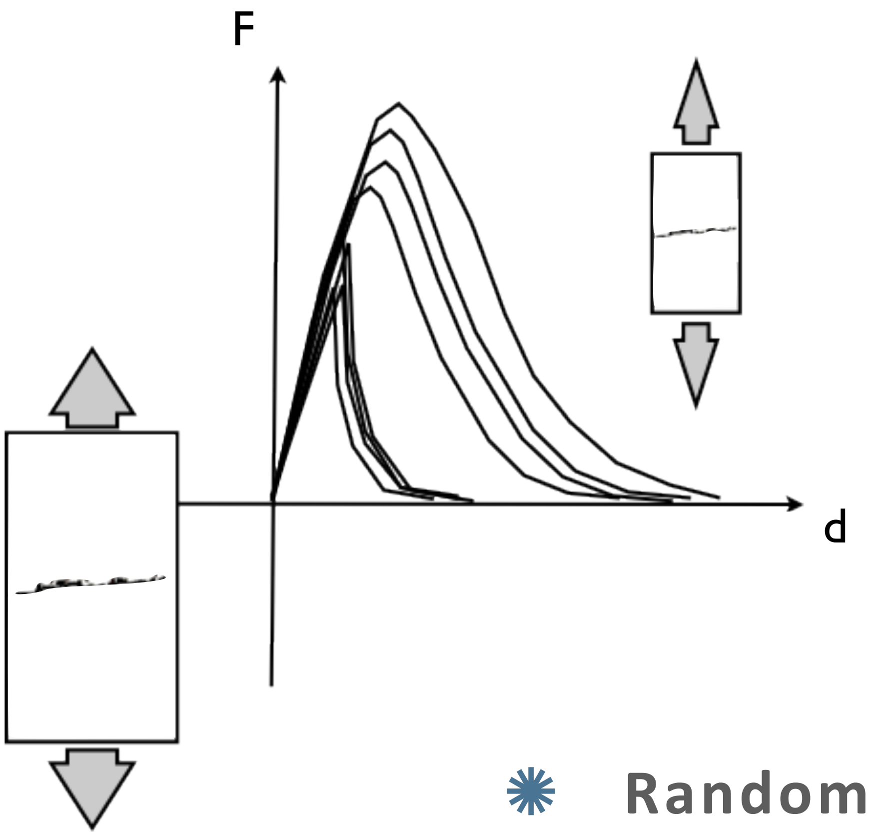
↓

Cracking processes at the material scale

↓

Macroscopic response of the stressed volume of material

CRACKING PROCESSES IN CONCRETE



- **Random behaviours**
- **Scale effects**

Heterogénéity & initial defects ramdomly distributed

↓

Cracking processes at the material scale

↓

Macroscopic response of the stressed volume of material

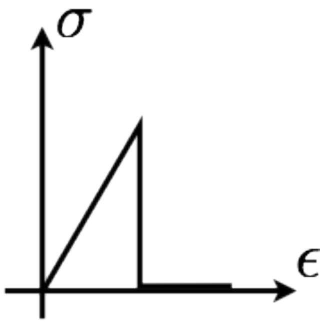
Basic idea:

Rossi [87], [92], [94]

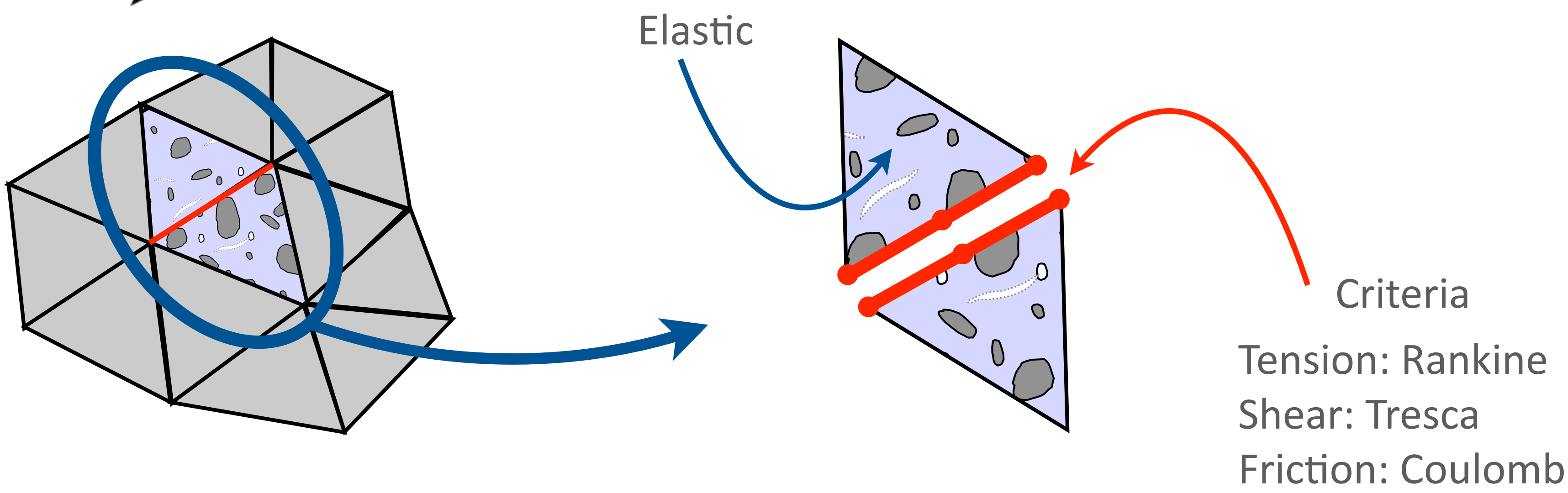
OBJECTIVES OF THE MODEL

- Explicit representation of cracks
- Accounting for heterogeneity and scale effects
- focused on a simple model with few input parameters

A PROBABILISTIC DISCRETE APPROACH



- **FEM + EXPLICIT REPRESENTATION OF CRACKS**
  use of contact elements

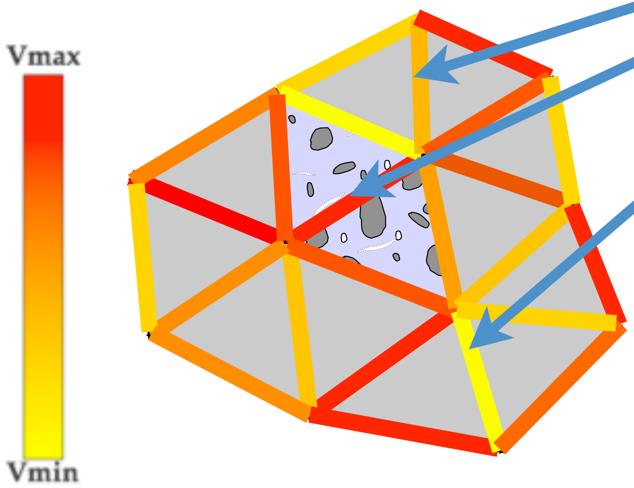


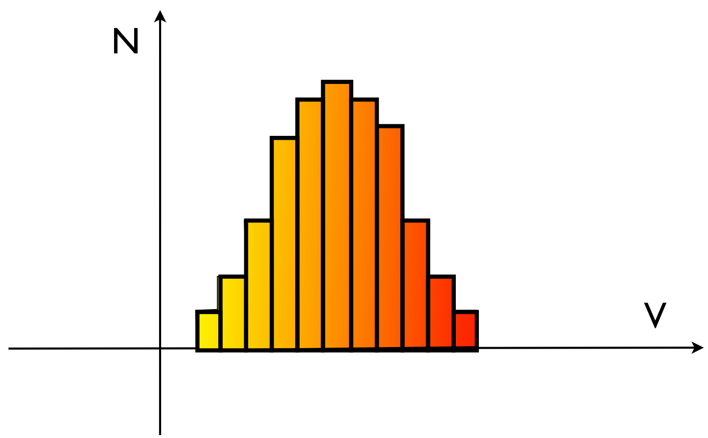
The size of the FE must be small compared to the size of the zone where stress gradients occur

A PROBABILISTIC DISCRETE APPROACH

HETEROGENEITY & SCALE EFFECTS

Mechanical properties
= random variables (Weibull law)





Physical mechanisms are considered to be the same whatever the size of the stressed volume

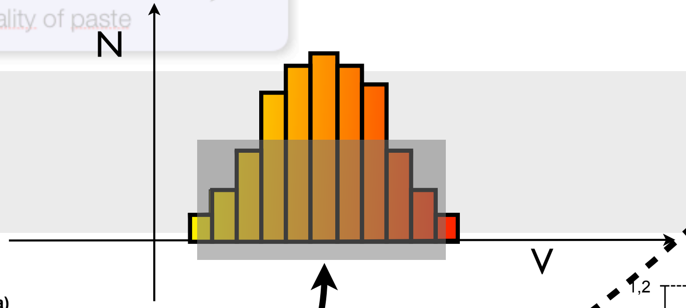
A PROBABILISTIC DISCRETE APPROACH

Rossi et al. 89 and 94

- FEW INPUT PARAMETERS

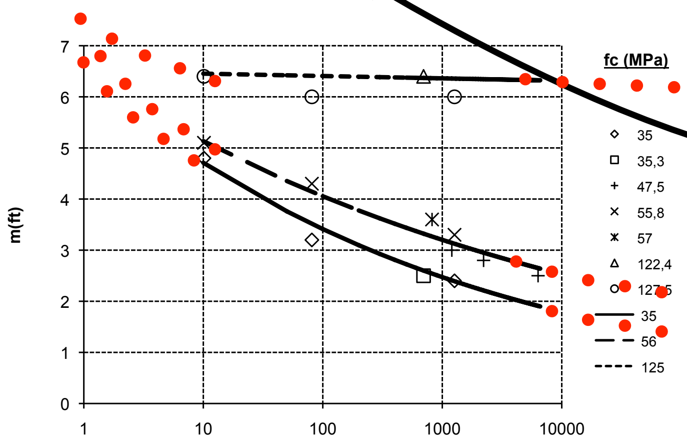
determined by inverse analysis

Tailhan [06]

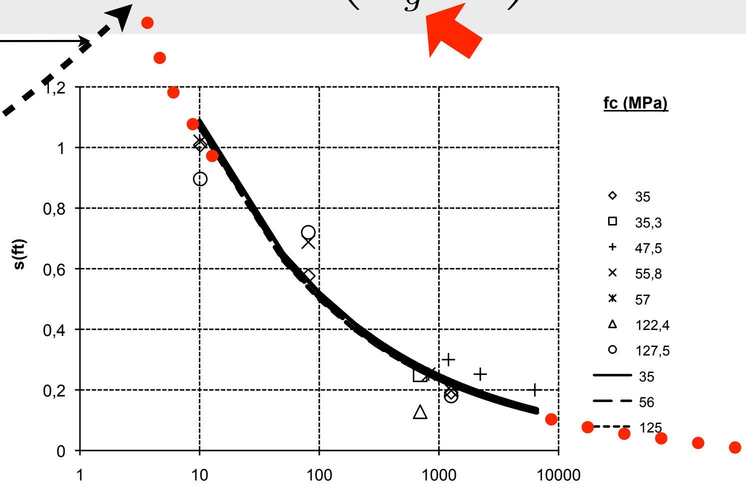


$$m(f_t) = f\left(\frac{V_m}{V_g}, f_c\right)$$

$$\sigma(f_t) = g\left(\frac{V_m}{V_g}, f_c\right)$$



$$\frac{V_m}{V_g}$$

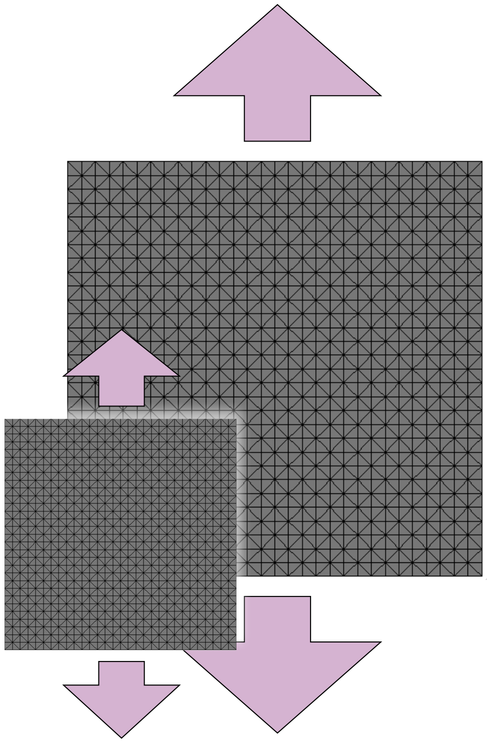


$$\frac{V_m}{V_g}$$

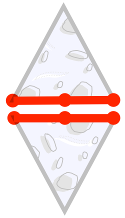
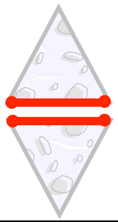
A PROBABILISTIC DISCRETE APPROACH

2D UNIAXIAL TENSION

10 simulations
same concrete (fc=50MPa, Dg=2cm)

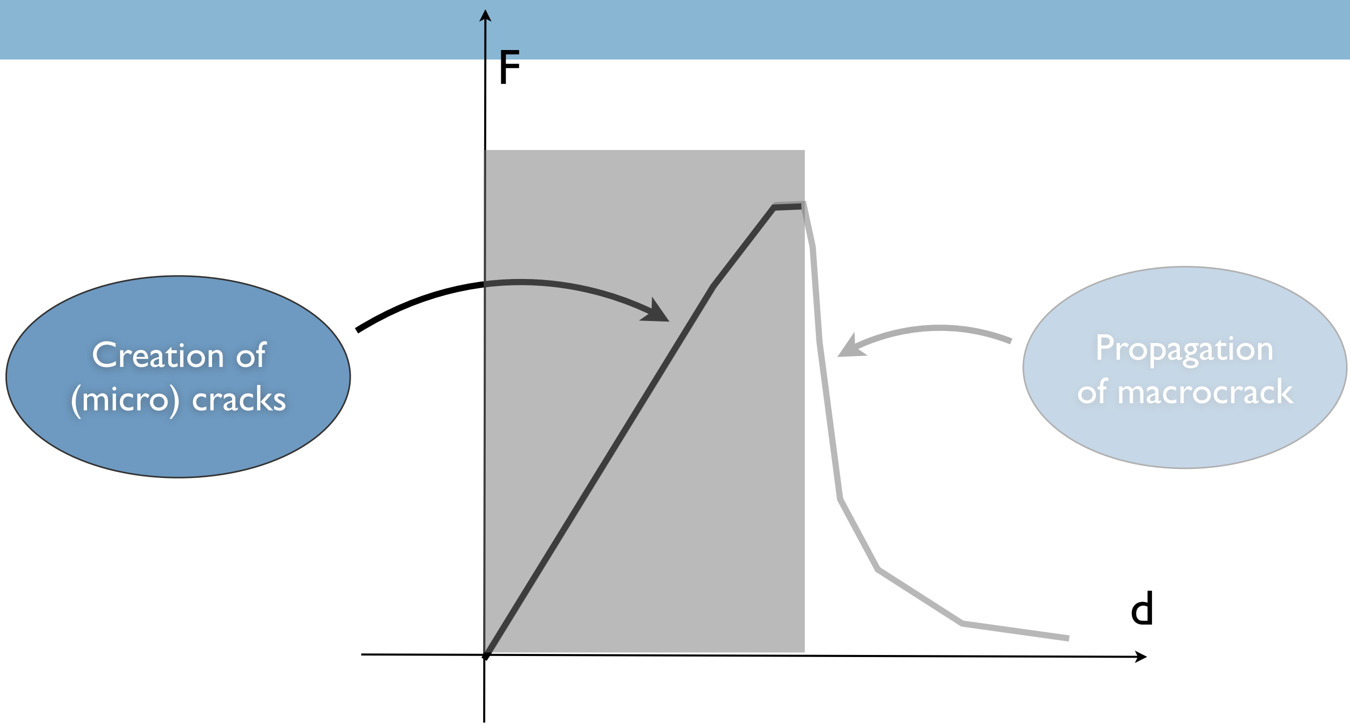


Tensile strength (mean val., sdt. dev.)

	V	10 x V
 Quad.	3.11 MPa 0.24 MPa	2.70 MPa 0.11 MPa
 Line.	3.5 MPa 0.34 MPa	2.5 MPa 0.36 MPa
Prediction	3.38 MPa 0.29 MPa	3 MPa 0.18 MPa

A PROBABILISTIC DISCRETE APPROACH

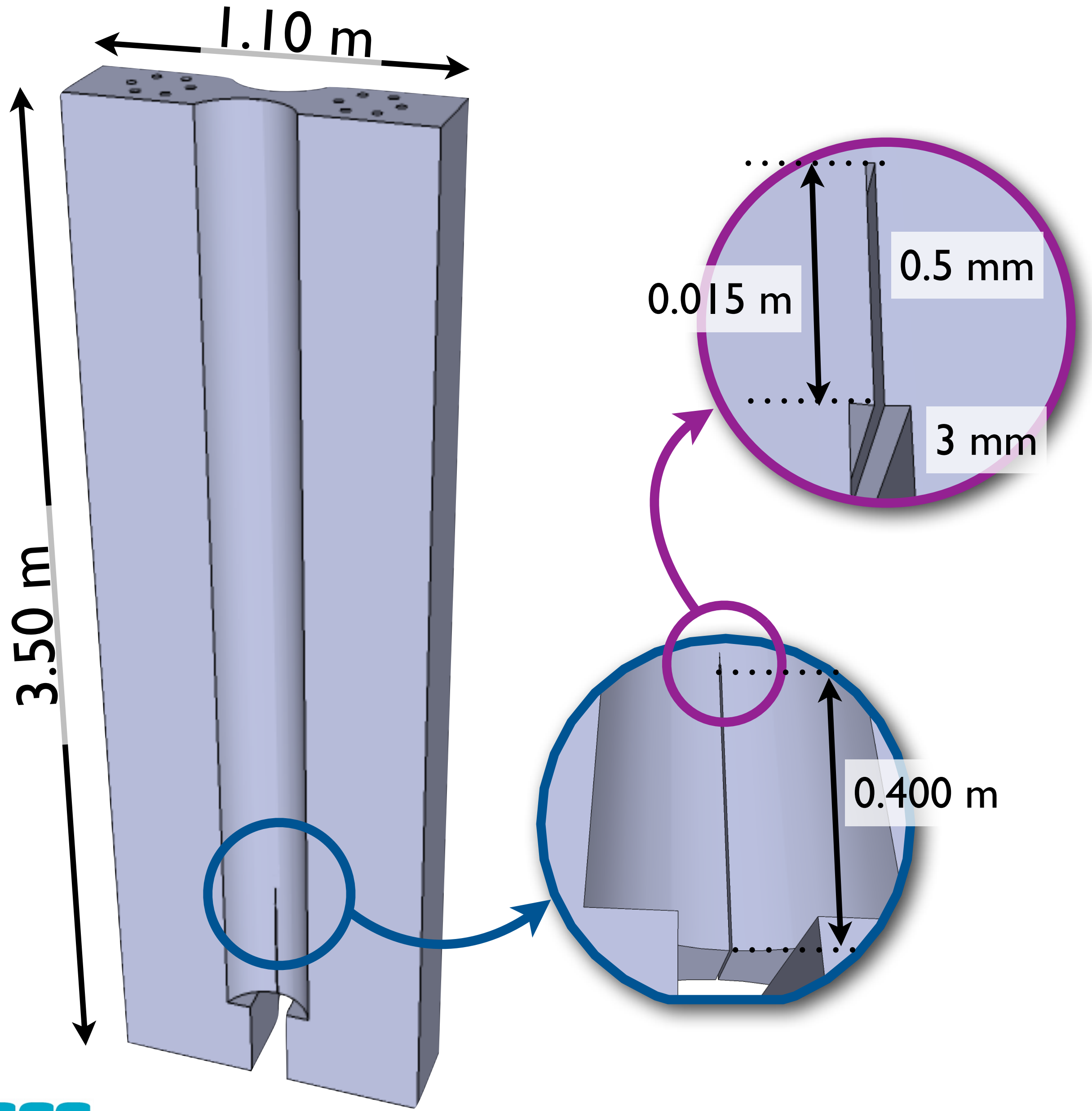
1rst CONCLUSION



As the model is rather based on the creation of crack planes in the material, it correctly represents the (material) behaviour up to the peak load

BUT: what about crack propagation ?

MACROCRACK PROPAGATION IN A DOUBLE CANTILEVER BEAM



TESTED STRUCTURE

Rossi [88]

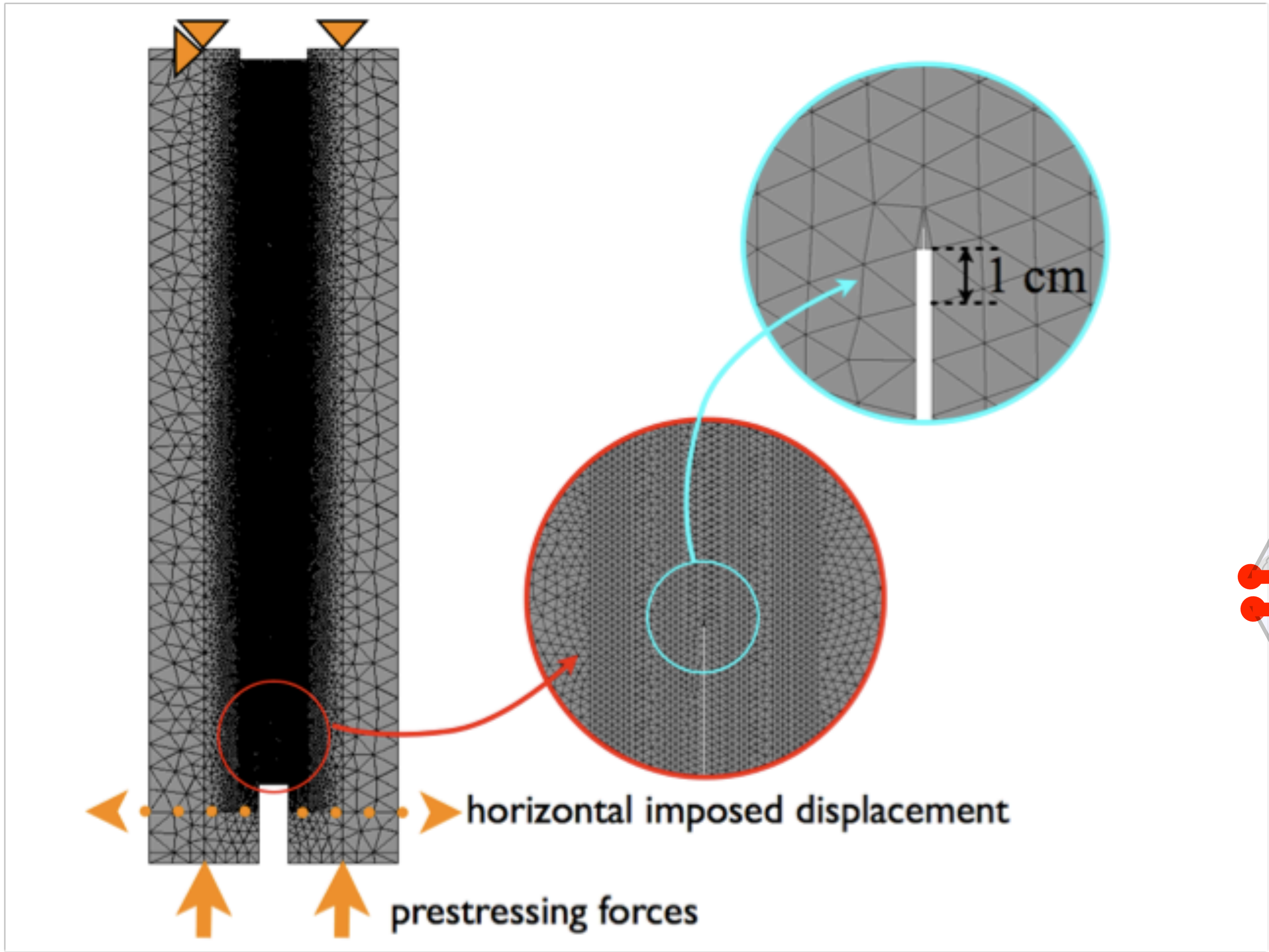
Concrete characteristics:

$$E = 35000MPa$$

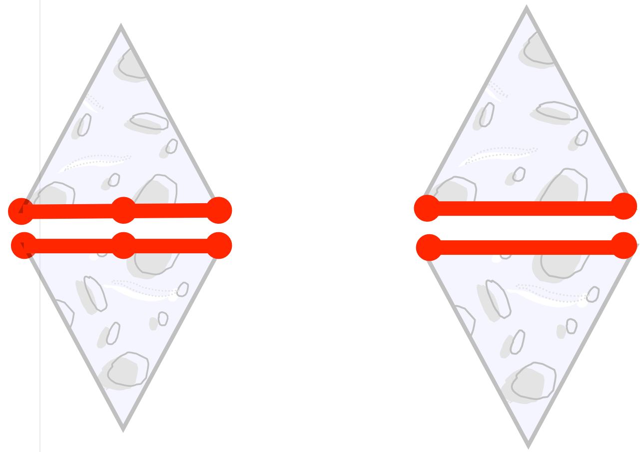
$$f_c = 50MPa$$

$$D_g = 12mm$$

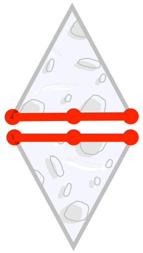
MACROCRACK PROPAGATION IN A DOUBLE CANTILEVER BEAM



2D SIMULATION (plane stresses)



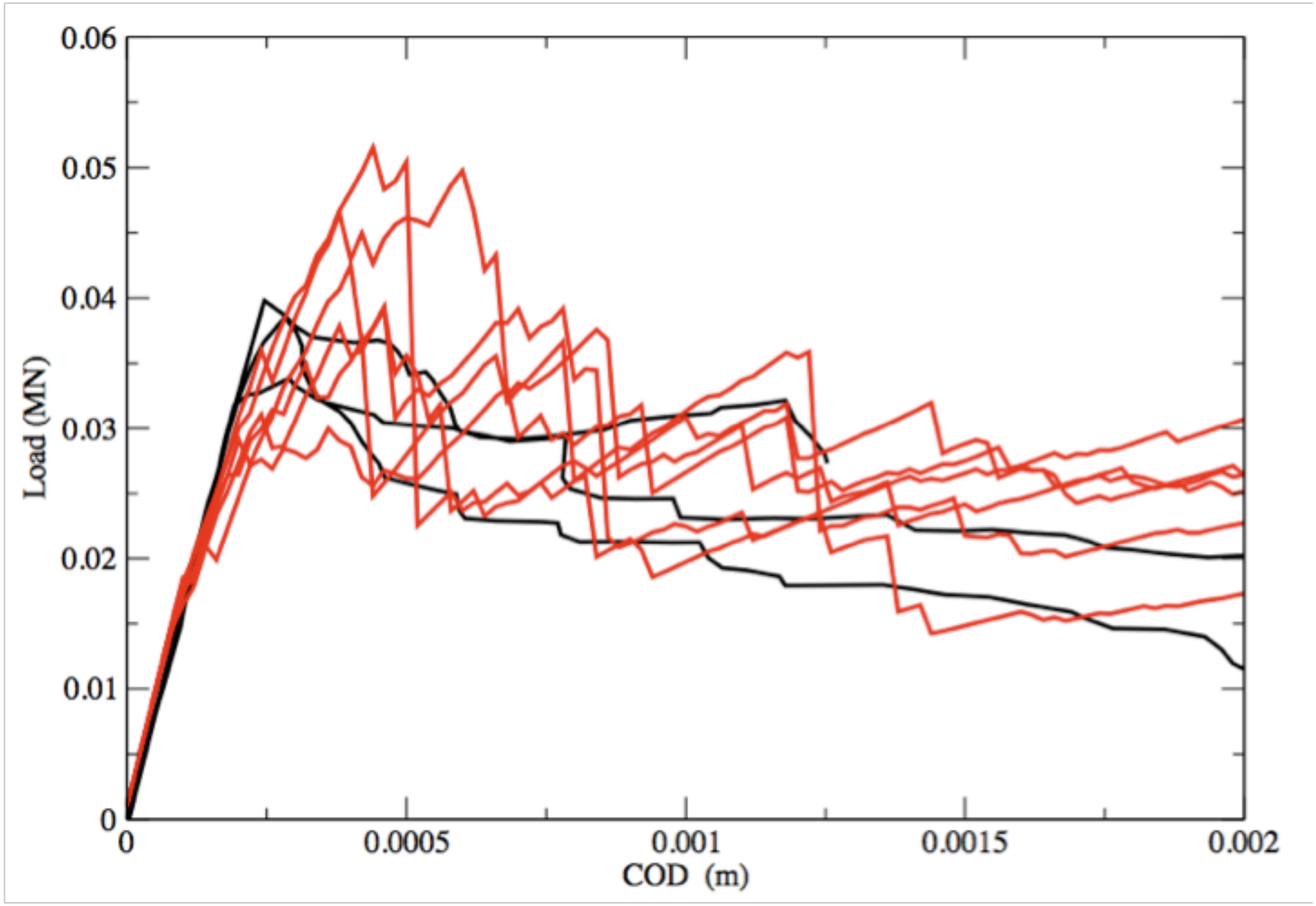
MACROCRACK PROPAGATION IN A DOUBLE CANTILEVER BEAM



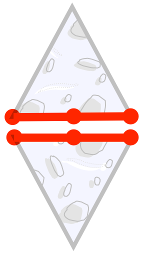
Quad.

RESULTS

3 tests - 6 simulations

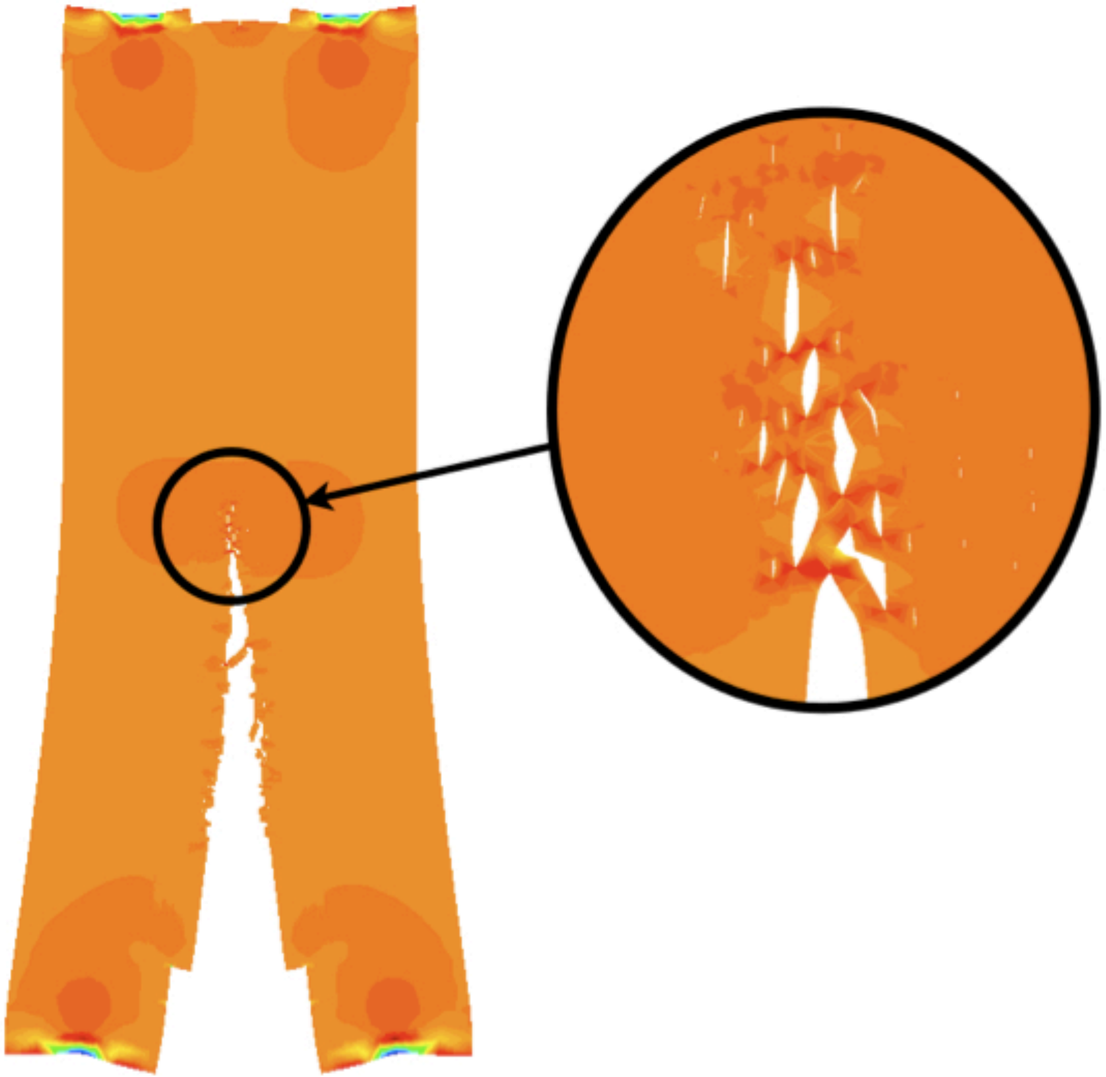


MACROCRACK PROPAGATION IN A DOUBLE CANTILEVER BEAM



Quad.

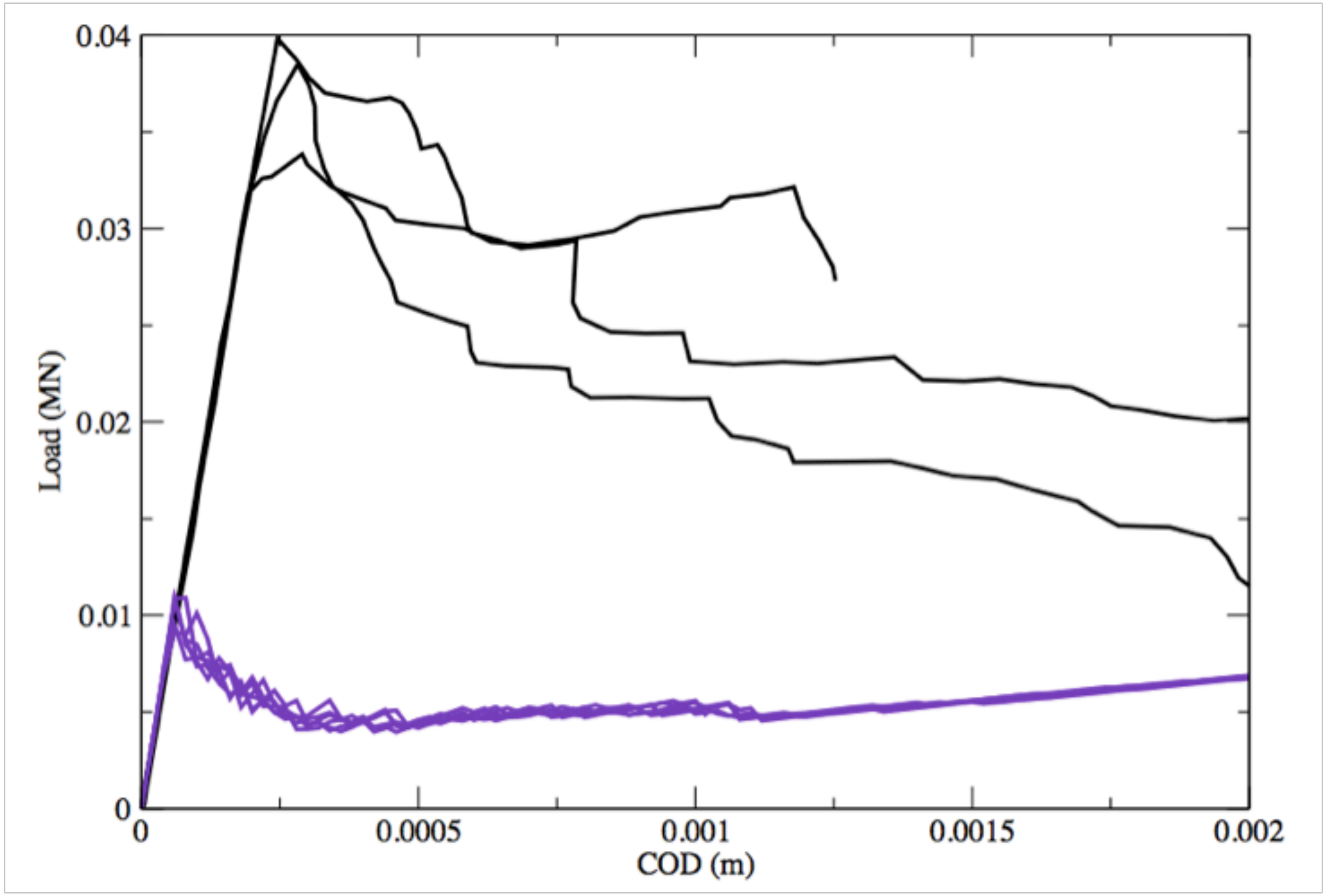
RESULTS



MACROCRACK PROPAGATION IN A DOUBLE CANTILEVER BEAM

Line.

RESULTS
3 tests - 6 simulations



MACROCRACK PROPAGATION IN A DOUBLE CANTILEVER BEAM

Line.

PROBLEM

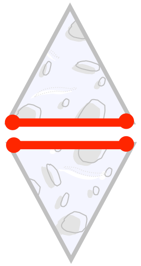
Why does the model fail with linear elements ?

The model does not represent crack propagation, it only approximates it by the successive creation of crack planes

- Failure criteria are computed at the centres of elements (numerical control of the crack creation)
- The linear element overestimates the stresses at its centre (poorness of the displacement description)

No adequation between the evaluation of stress and the criterion that controls the crack propagation

MACROCRACK PROPAGATION IN A DOUBLE CANTILEVER BEAM



Line.

ADAPTATION OF THE MODEL

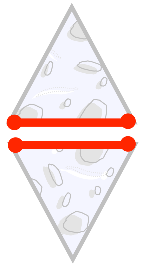
(for the linear element)

Basic idea

The creation of the crack plane at the scale of the element is the consequence of the coalescence and the propagation of (micro) cracks at a smaller scale.

☞ local dissipation of a given amount of energy.

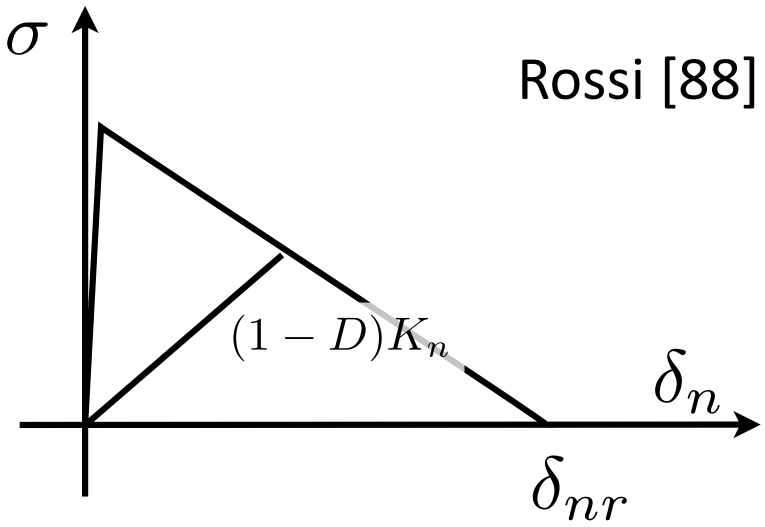
MACROCRACK PROPAGATION IN A DOUBLE CANTILEVER BEAM



Line.

ADAPTATION OF THE MODEL
(for the linear element)

Simple damage model



From experimental results :

Rossi [88]

$$K_{IC} = \sqrt{EG_C} \quad \text{(plane stresses)}$$

$$K_{IC} = 2.16\ MPa.\sqrt{m}$$

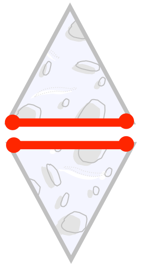
$$E = 35000 \quad MPa$$

$$G_C = 1.31\,10^{-4}\ MPa.m$$

(mean value)

random variable

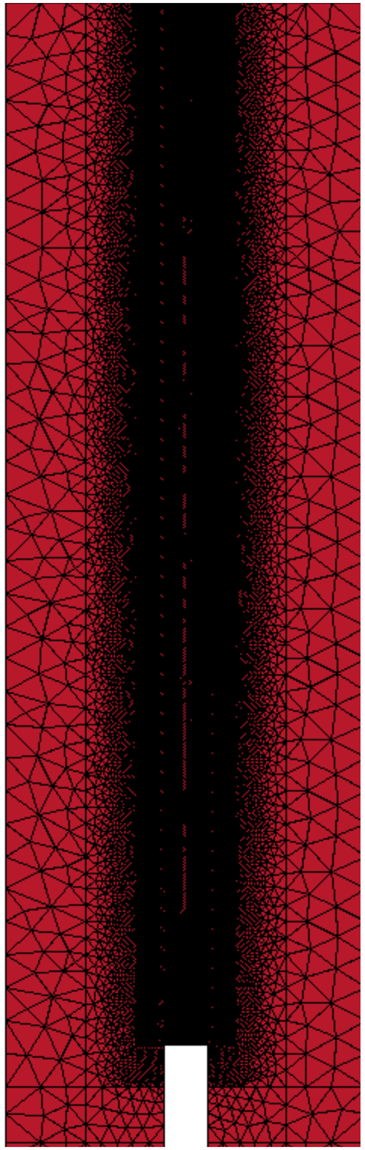
MACROCRACK PROPAGATION IN A DOUBLE CANTILEVER BEAM

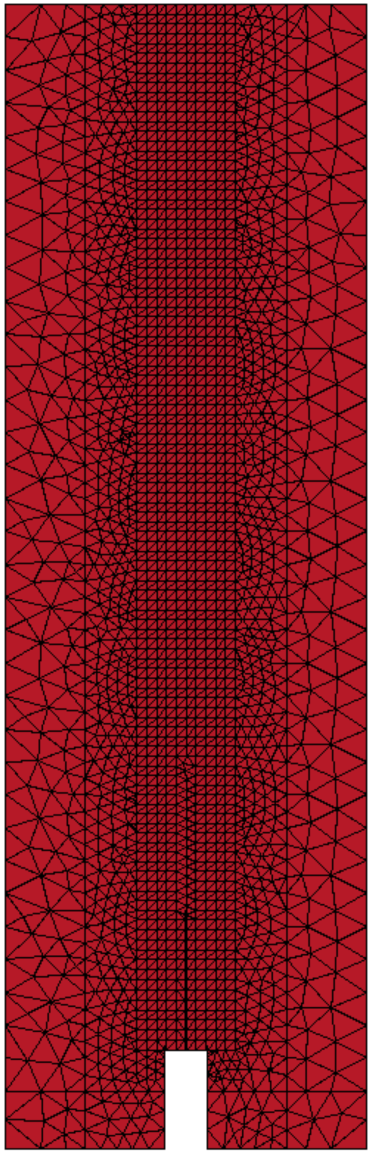


Line.

2D SIMULATIONS

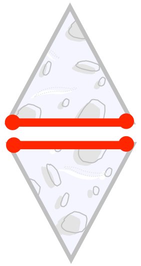
Fine





Coarse

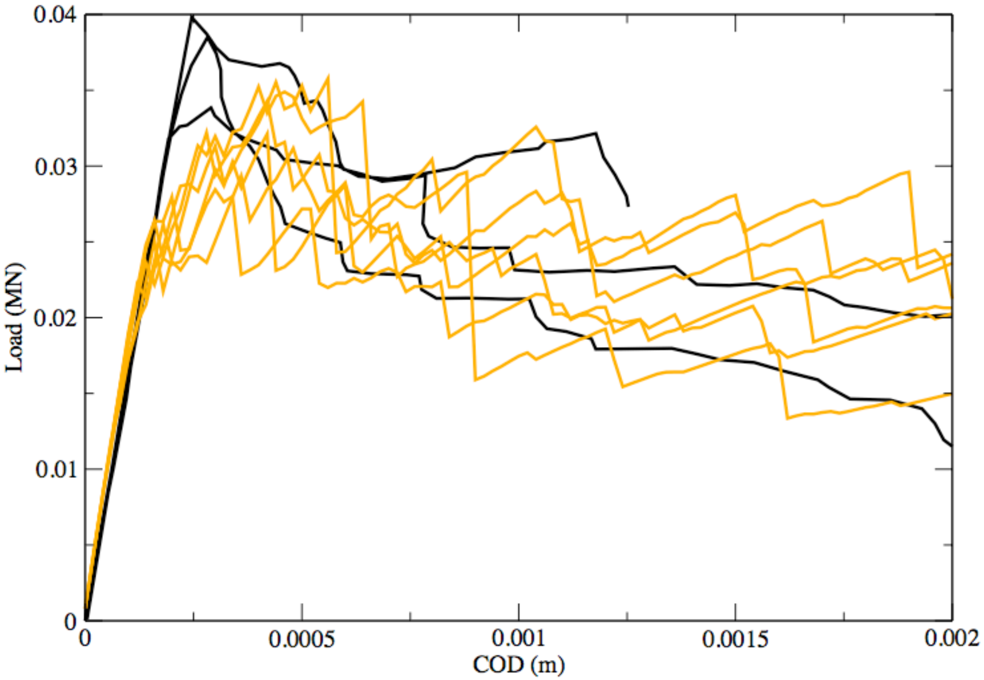
MACROCRACK PROPAGATION IN A DOUBLE CANTILEVER BEAM



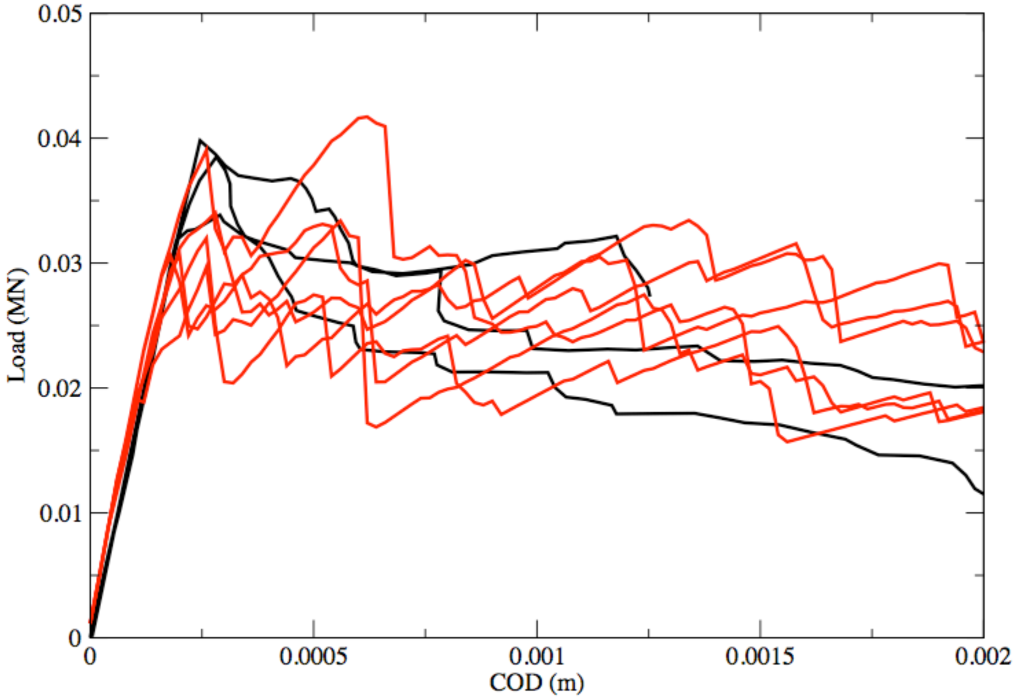
Line.

RESULTS
3 tests - 6 simulations

Fine mesh



Coarse mesh



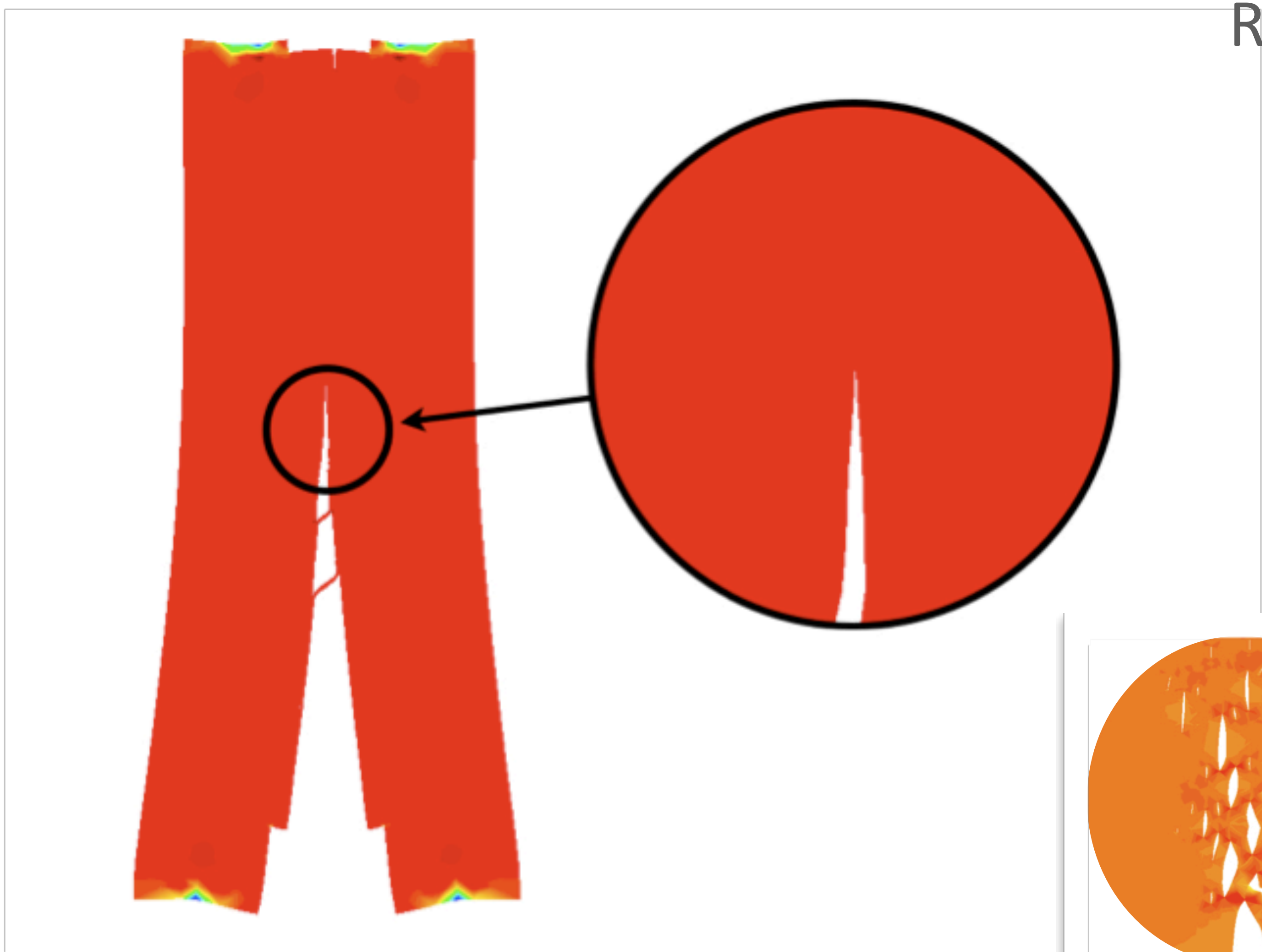
$$m(G_c) = 1.31\,10^{-4}\;MPa.m$$

$$\frac{\sigma}{m}(G_c) = 5 \qquad \frac{\sigma}{m}(G_c) = 1$$

MACROCRACK PROPAGATION IN A DOUBLE CANTILEVER BEAM

Line.

RESULTS



CONCLUSIONS: MODELS AND SCALES OF MODELLING

- When only crack creation is involved in the problem probabilistic elastic brittle behaviour with both quadratic or linear elements are efficient

- When crack propagation is the driving physical mecanism of the problem, both:
  - probabilistic elastic brittle behaviour with quadratic elements
  - or probabilistic damage behaviour with linear elements

  can be used

  but they don't act at the same scale

CONCLUSIONS: MODELS AND SCALES OF MODELLING