SSCS 2012
Numerical Modeling
Strategies for Sustainable Concrete Structures
Aix en Provence - France
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AFGC

A RC fibre beam element for full modelling of the bending-shear response by dual section approach

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Motivation

L'Aquila earthquake, April 6, 2009

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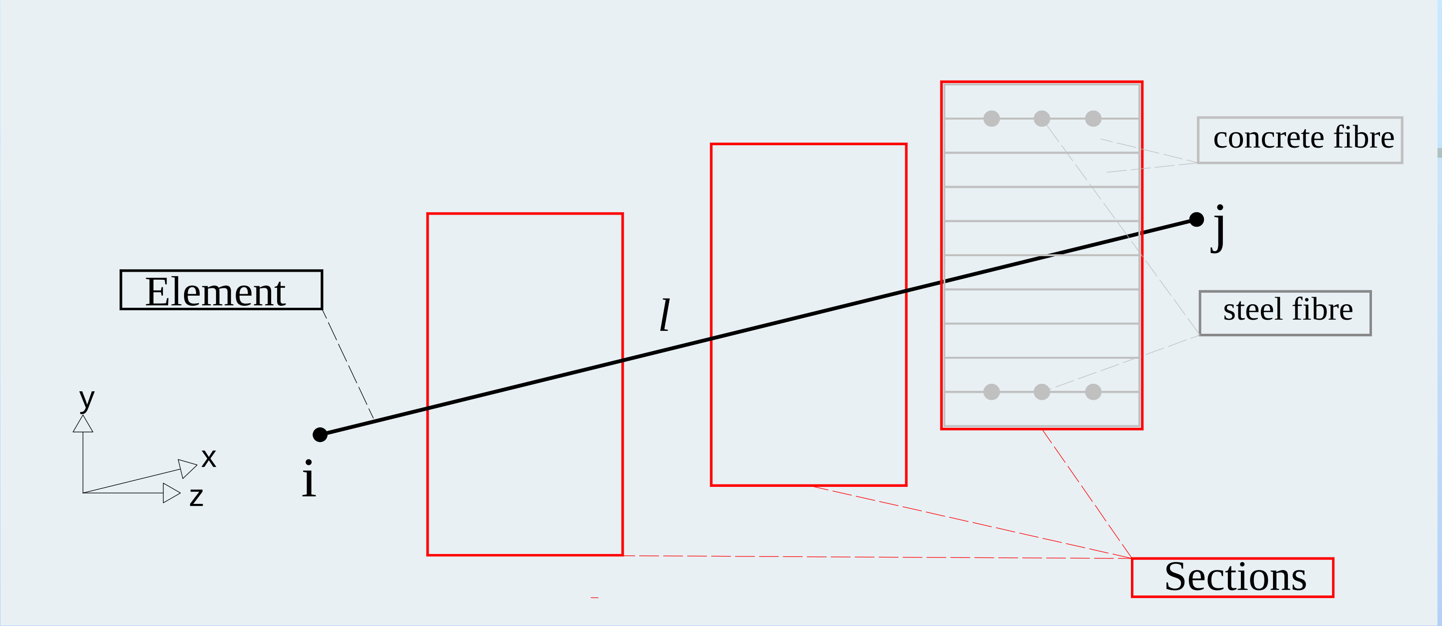
Motivation

- Develop modeling capabilities for assessing seismic vulnerability of existing RC Structures
- Nonlinear methods of analysis
  - Static Pushover and Nonlinear Dynamic
- Concentrate on frame approach
- Force-based elements
  - "Exact" Euler-Bernoulli Beam Element (fiber section)
    - Rome and Berkeley schools (Ciampi, Spacone, Filippou, …, OpenSees, …) – commercial software
  - "Exact" **Timoschenko Element** (fiber section)
    - Rome school (Petrangeli, Ciampi, Pinto)

The beam finite element



1) Element

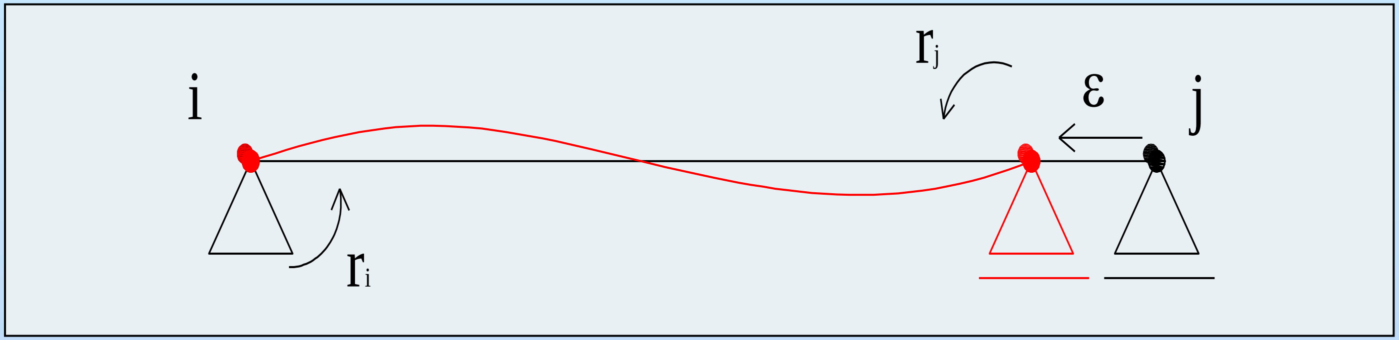
2) Section

3) Fibre

- **steel** **uniaxial constitutive law**
- **concrete** **biaxial constitutive law**

(microplane model – Petrangeli and Ozbolt)

Element force recovery (state determination)



IN

Element deformations

$$Q = [\varepsilon; r_i; r_j]^T$$

Nodal forces

$$P = [N; M_i; M_j]^T$$

OUT

Element Iterative solution based on beam equilibrium

Element state determination

Iterative solution based beam equilibrium

Element deformations

$$Q=[\varepsilon;r_i\ ;r_j\ ]^T$$

Generalised deformations (section)

$$q(x)=[\varepsilon(x);\phi(x);\gamma(x)]^T$$

$$Q=\int_l b^T(x)\cdot q(x)dx$$

Nodal forces

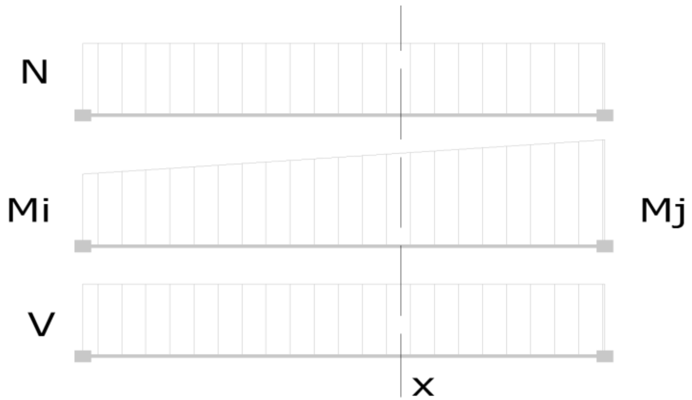
$$P=[N;M_i\ ;M_j\ ]^T$$

Generalised stresses

$$p(x)=[N(x);M(x);V(x)]^T$$

$$p(x)=b(x)\cdot P^e$$

b(x): stress shape functions matrix (right)



$$M(x)=M_i+(M_j-M_i)\frac{x}{l}$$

$$V(x)=\frac{M_j+M_i}{l}$$

$$N(x)=N$$

Element state determination

Iterative solution based beam equilibrium

$$\Delta q_i(x) = \Delta q_0(x) + \sum_i rq_i(x)$$

particular solution *homogeneous solutions*

$$\Delta Q = \int_l b(x)^T \Delta q_0(x) dx$$

boundary condition

$$rp_i(x) = b(x)\Delta P_i - \Delta p_{i-1}(x)$$

$b(x)\Delta P_i$ *by beam equilibrium*

$\Delta p_{i-1}(\Delta q_i)$ *by constitutive laws*

therefore corrective terms of deformations field are:

$$rq_i(x) = f(x) rp_i(x)$$

f(x): section flexibility matrix

Element state determination

Iterative solution based beam equilibrium

ΔQ

$\Delta q_0(x)$

LC

$\Delta p_0(x)$

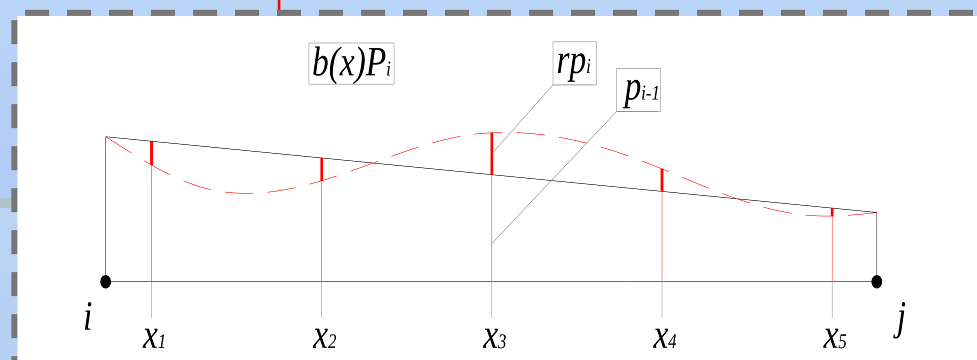
beam integration

$\Delta P_1(x)$

beam equilibrium

$rp_1(x) = b(x)\Delta P_1 - \Delta p_0$

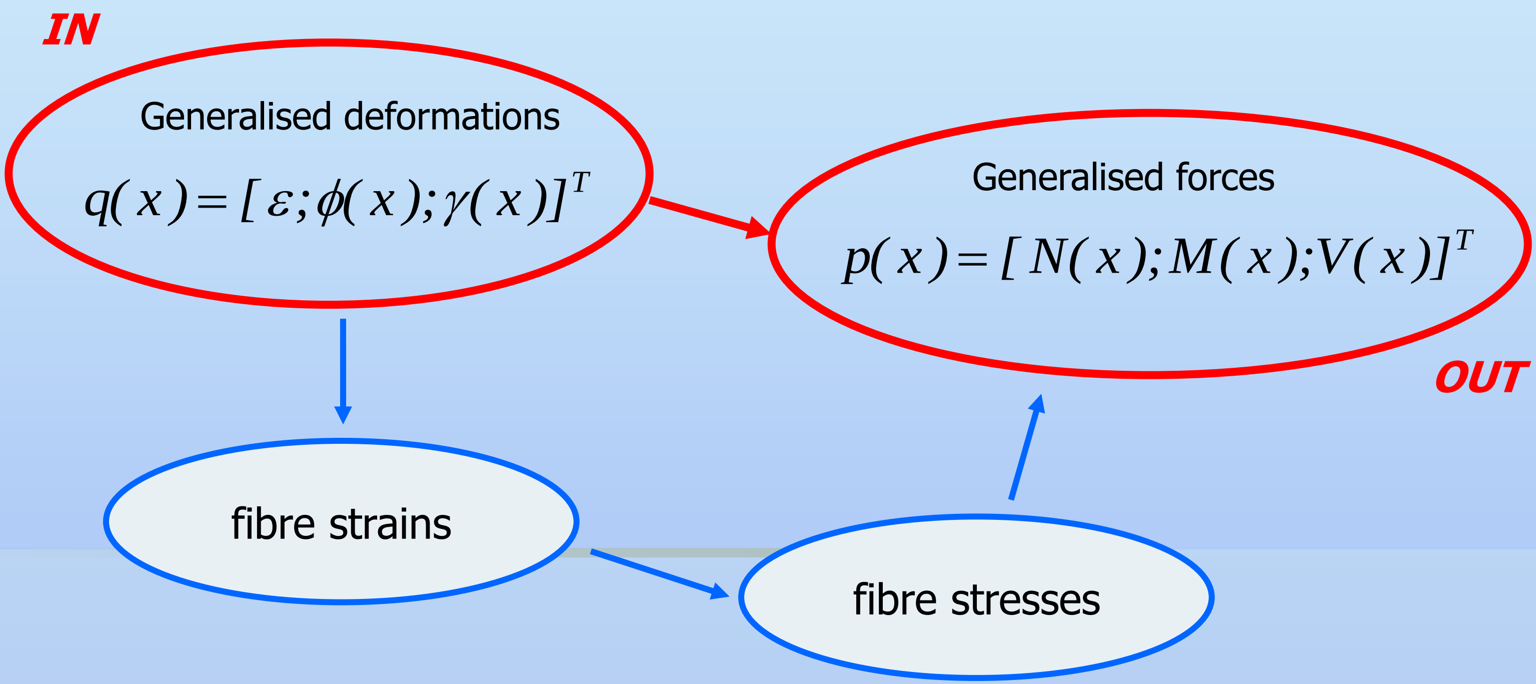
check convergence



corrective term of deformation field

$rq_1(x) = f(x) rp_1(x)$

Section state determination



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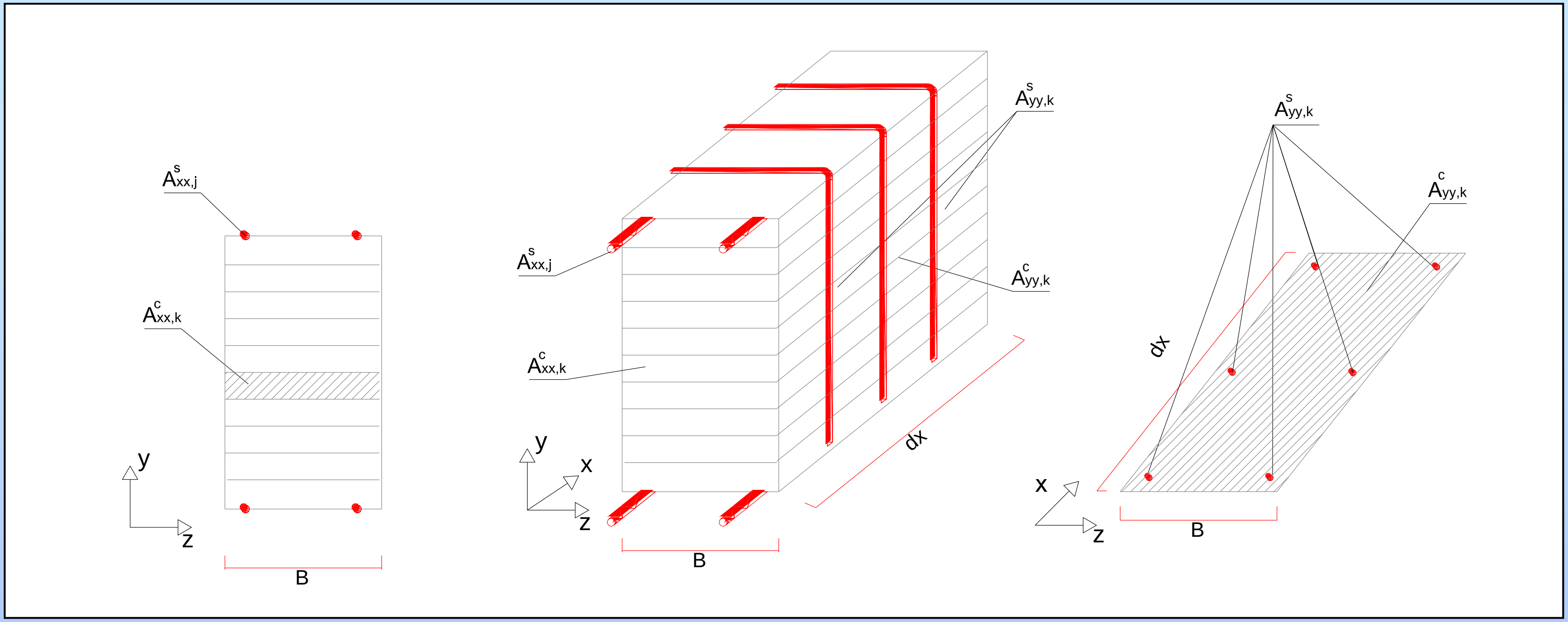
Section state determination



The steel fibre is characterized:
- Normal area ($A^s xx$)

$$\sigma^s_{xx} = f(\varepsilon^s_{xx})$$

The concrete fibre is characterized:
- Normal area ($A^c xx$)
- Lateral area ($A^c yy$)
- Ratio lateral reinforcement (ρyy)

$$\sigma^c = [\sigma^c_{xx};\sigma^c_{yy};\tau^c_{xy}]$$

$$\varepsilon^c = [\varepsilon^c_{xx};\varepsilon^c_{yy};\varepsilon^c_{xy}]$$

Section state determination

Generalised deformations

$$q(x) = [\varepsilon(x), \phi(x), \gamma(x)]$$

Concrete fibre strains:

- axial strain *(also fibre steel – perfect bond)*

$$\varepsilon^{c,s}_{xx,k}(x) = \varepsilon(x) + \phi(x)\, y^{c,s}_k(x) \qquad k = 1 \ldots n_{fc} \qquad k = 1 \ldots n_{fs}$$

(hp. plane sections)

- lateral strain

$$\sigma^{c}_{yy,k} + \rho_{yy,k}\, f(\varepsilon^{c}_{yy,k}) = 0 \qquad k = 1 \ldots n_{fc}$$

(fibre equilibrium lateral reinforcement and concrete)

iterative solution $\varepsilon^{c}_{yy,k} = \varepsilon^{c,0}_{yy,k} + \varepsilon^{c,1}_{yy,k} + \ldots + \varepsilon^{c,i}_{yy,k}$

Section state determination

Concrete fibre strains:

- shear strain (DUAL SECTION APPROACH)

This approach is based on the beam differential equilibrium equation

$$\frac{dM}{dx} = V$$

When this eq. is applied in the beam element with discrete fibre section

$$F_{xy,k} = \left(F^{c}_{xx,1-k}(x_{s+1}) + F^{s}_{xx,1-k}(x_{s+1}) \right) - \left(F^{c}_{xx,1-k}(x_{s}) + F^{s}_{xx,1-k}(x_{s}) \right)$$

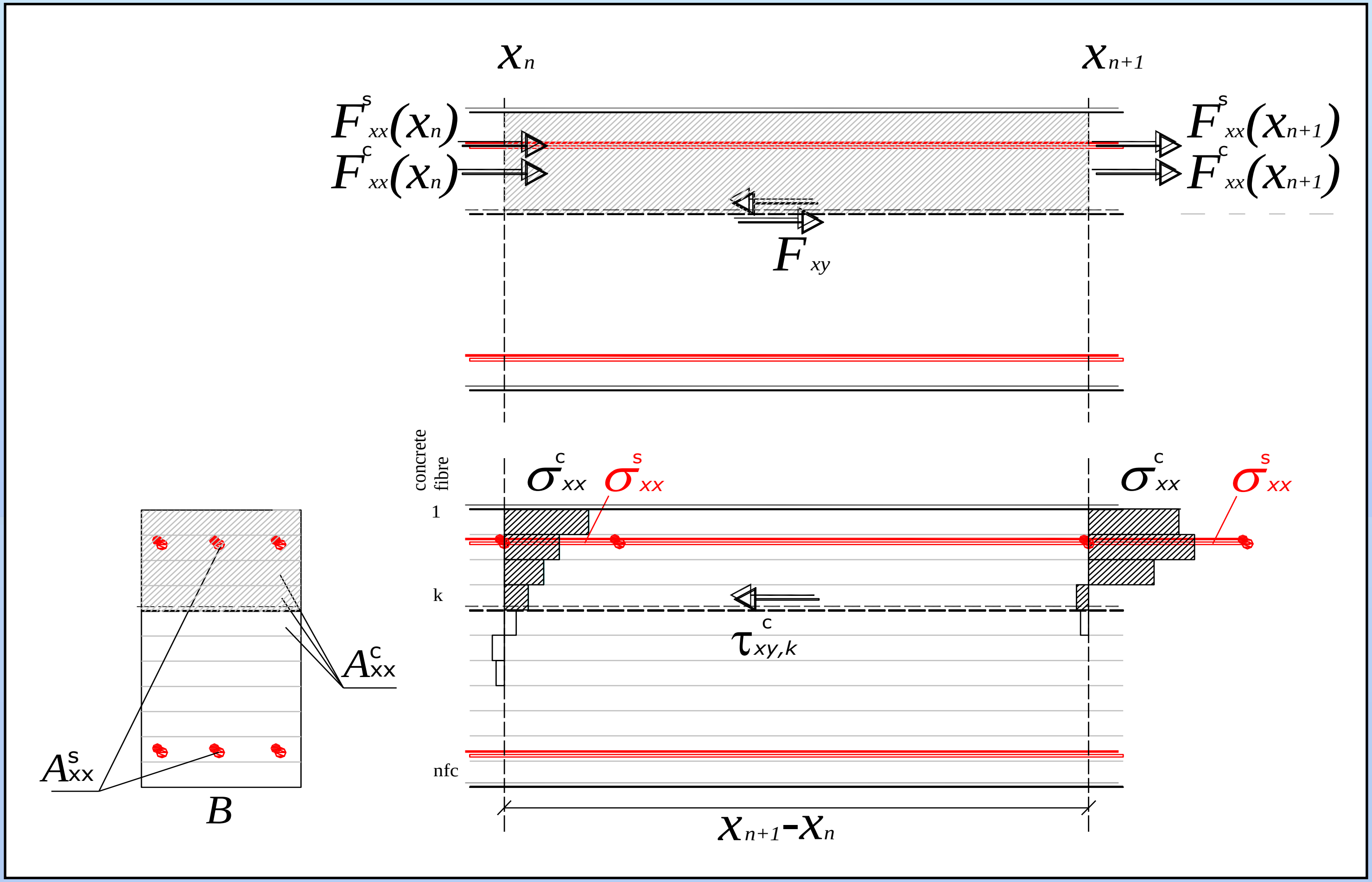
Re-arranging in the stress fibre

$$\tau^{c}_{xy,k}\big|_{DS} = \frac{\sum_{1}^{k} \sigma^{c}_{xx,k}(x_{n+1}) A^{c}_{xx,k} - \sum_{1}^{k} \sigma^{s}_{xx,k}(x_{n+1}) A^{s}_{xx,k}}{B \cdot (x_{n+1} - x_{n})} - \frac{\sum_{1}^{k} \sigma^{c}_{xx,k}(x_{n}) A^{c}_{xx,k} - \sum_{1}^{k} \sigma^{s}_{xx,k}(x_{n}) A^{s}_{xx,k}}{B \cdot (x_{n+1} - x_{n})}$$

Section state determination

Concrete fibre strains:

- shear strain (DUAL SECTION APPROACH)



Section state determination

Concrete fibre strains:

- shear strain (DUAL SECTION APPROACH)

iterative solution

$$\varepsilon^{c}_{xy,k} = g(y_k)\gamma(x) + \left(r\varepsilon^{c,1}_{xy,k} - g(y_k)R^{1}_{\gamma}(x)\right) + \ldots + \left(r\varepsilon^{c,i}_{xy,k} - g(y_k)R^{i}_{\gamma}(x)\right)$$

particular term

homogeneous corrections

$g(y_k)$ shape distribution shear strain along the section

respect of boundary condition

$$\gamma(x) = \frac{\sum_{k=1}^{nfc} \varepsilon^{c}_{xy,k}(x) A^{c}_{xx,k}}{\sum_{k=1}^{nfc} A^{c}_{xx,k}}$$

Section state determination

Concrete fibre strains:

- shear strain (DUAL SECTION APPROACH)

iterative solution

$$\varepsilon^{c}_{xy,k} = g(y_k)\gamma(x) + \left(r\varepsilon^{c,1}_{xy,k} - g(y_k)R^{1}_{\gamma}(x)\right) + \ldots + \left(r\varepsilon^{c,i}_{xy,k} - g(y_k)R^{i}_{\gamma}(x)\right)$$

homogeneous corrections

$$\tau^{c}_{xy,k} - \tau^{c}_{xy,k}\big|_{DS} = r\tau^{c}_{xy,k}$$

where $\tau^{c}_{xy,k}$ shear stress found an incorrect strain deformation

$$r\varepsilon^{c}_{xy,k} = \frac{r\tau^{c}_{xy,k}}{G}$$

$$R_{\gamma}(x) = \frac{\sum_{k=1}^{nfc} r\varepsilon^{c}_{xy,k} A^{c}_{xx,k}}{\sum_{k=1}^{nfc} A^{c}_{xx,k}}$$

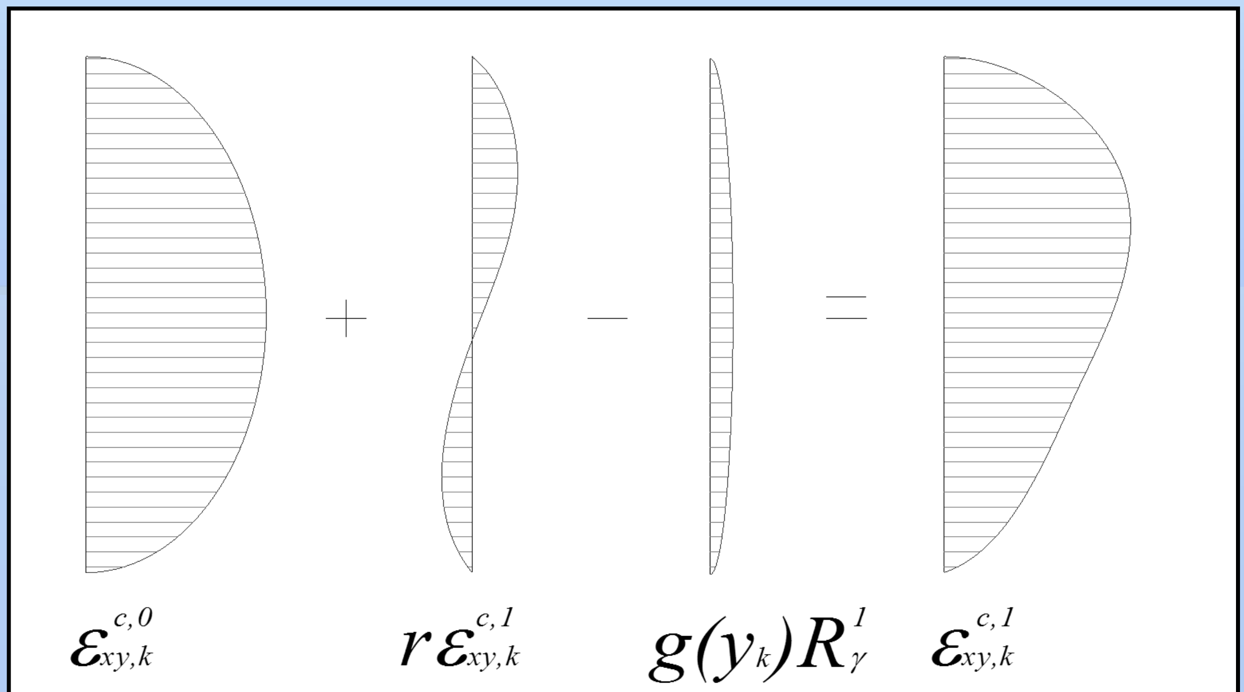
Section state determination

Concrete fibre strains:

- shear strain (DUAL SECTION APPROACH)

iterative solution

$$\varepsilon^{c}_{xy,k} = g(y_k)\gamma(x) + \left(r\varepsilon^{c,1}_{xy,k} - g(y_k)R^{1}_{\gamma}(x)\right) + \ldots + \left(r\varepsilon^{c,i}_{xy,k} - g(y_k)R^{i}_{\gamma}(x)\right)$$



TESTS: short bridge piers

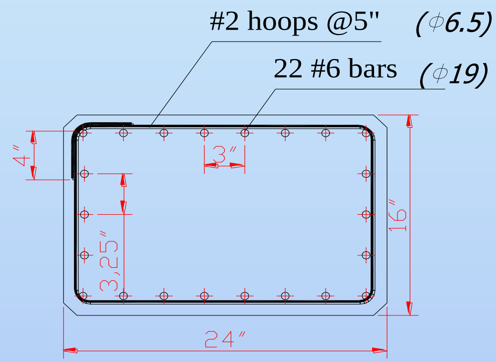
University of California San Diego

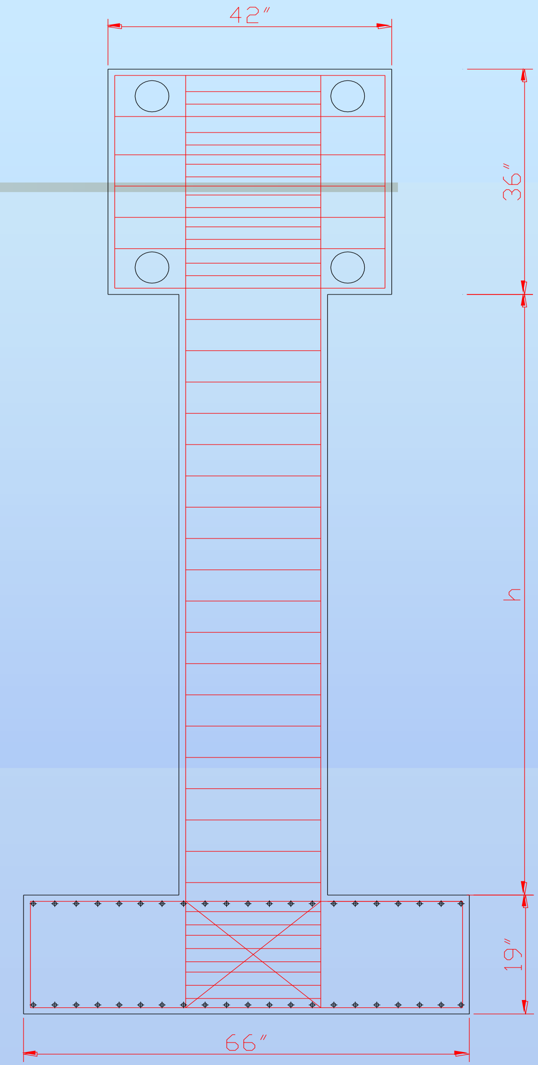
Yan Xiao, 1993

"Steel Jacket Retrofit For Enhancing Shear Strength Of Short Rectangular Reinforced Columns".

$$A_{sw}/s = 500\,mm^2/m$$

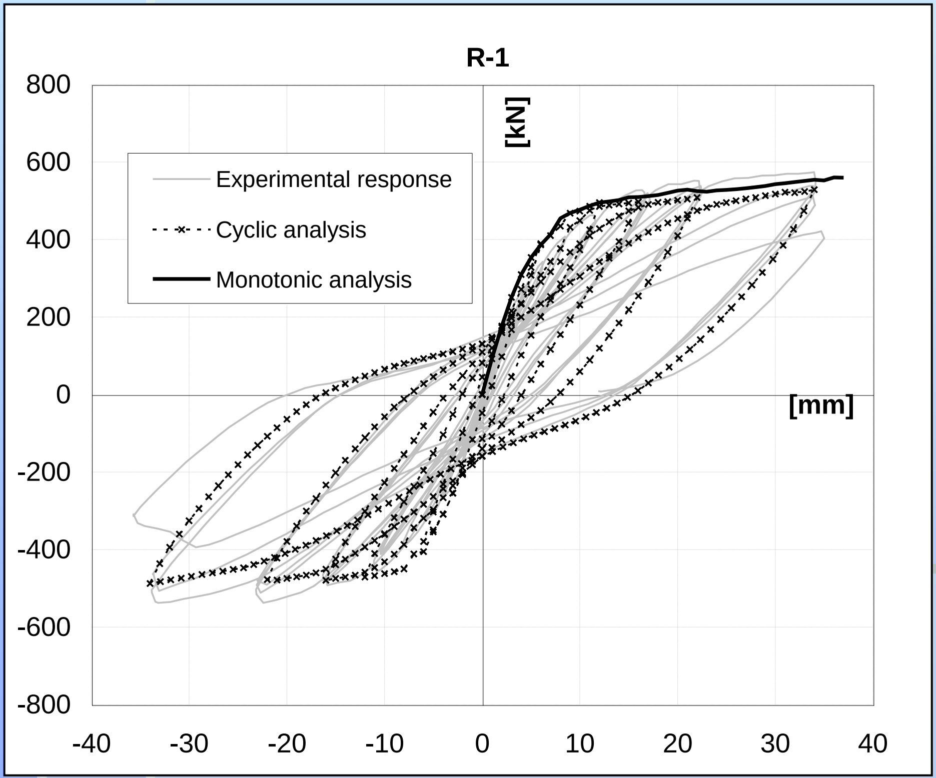
$$\sigma_N / f_c \cong 5\%$$

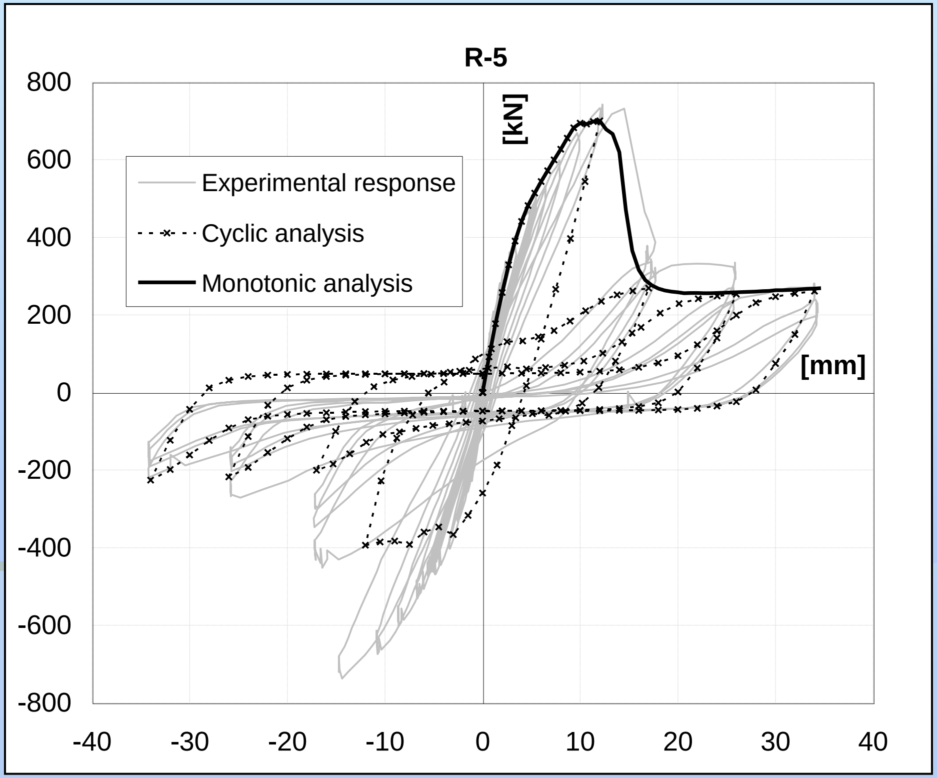




	R-1	R-5
steel ratio (%)	2.5	2.5
aspect ratio (-)	2	1.5
compression (MPa)	-2.1	-2.1
f_{cc} (MPa)	-37.9	-32.7
$f_{y,bars}$ (MPa)	317	469
$f_{y,hoops}$ (MPa)	360	324

TESTS: short bridge piers

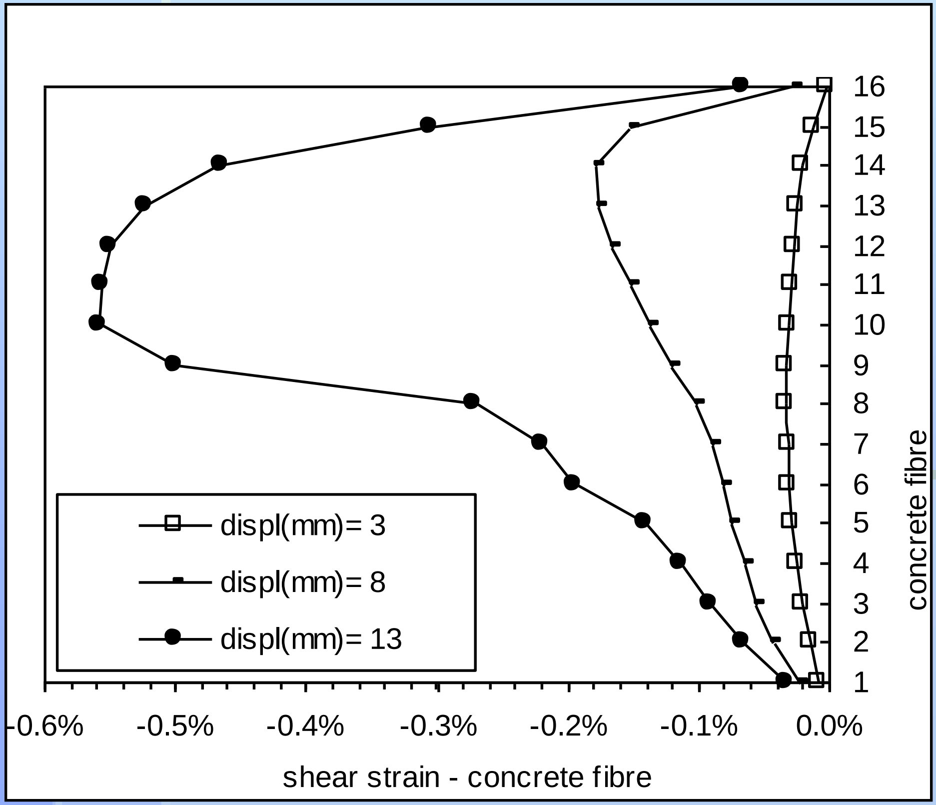


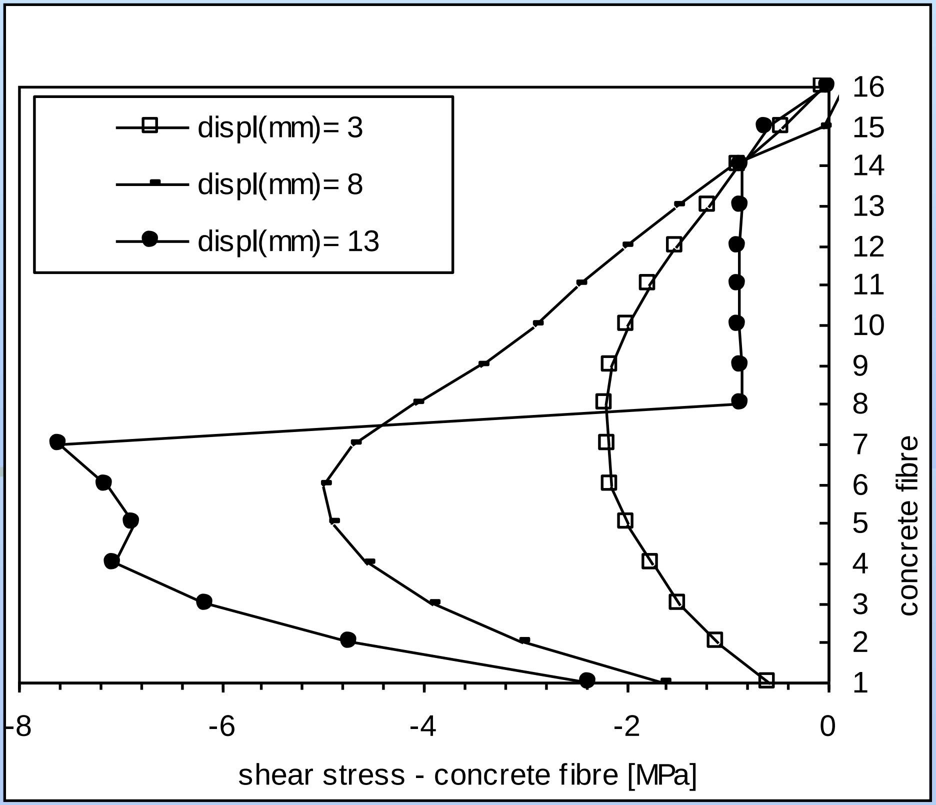


TESTS: short bridge piers

Evolution of concrete fibre shear stress-strain by dual section approach

Column R-5





thank you for your attention