Finite differences or volumes on grid, an alternative numerical approach

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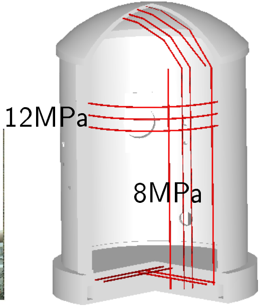
Outline

Pre-stressed concrete of containment vessels



Tensions in cables induce a compressive stress in concrete

- Good mechanical behaviour in case of internal pressure
- air tightness

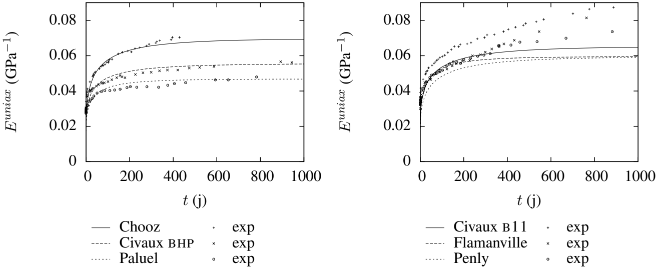
Creep in concrete hinders prestress forces.

Creep in concrete

Direct experiment is difficult :

- many concretes
- Time scale
- Complexity of the phenomenon

Taking account of lower scales may improve our knowlegde of the viscoelastic behaviour of concrete.

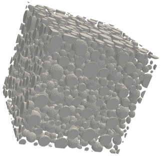


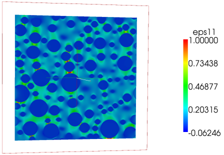
Comparison between models and experimental mesures on 6 concretes.

Modélisation micro-macro du fluage propre du béton
Julien Sanahuja et al. ,19 ème Congrès Français de Mécanique 2009

Full-field methods in homogeneization

Input : a 3D image of the microstructure
and the behaviour of each phase (cement paste, gravel)
Output : The macroscopic behaviour of the material (concrete)



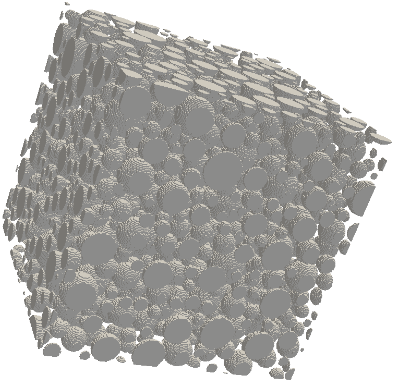


To solve the local problem, finite differences/volumes are used on grids.

- Systematic $\Rightarrow$ Useful if many runs are needed (statistics)
- Quality $\Leftrightarrow$ high resolution
- Models can handle larger numbers of degrees of freedom (FFT)

B11

2024 non-overlapping spherical inclusions with various radius, non periodic because of edges.
The volumic ratio of gravel is 0.4018.



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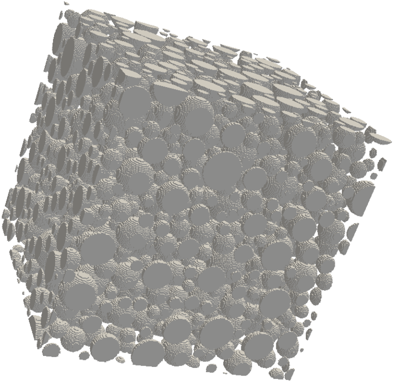
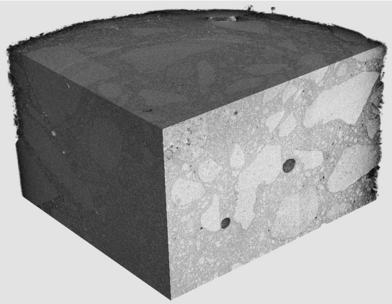
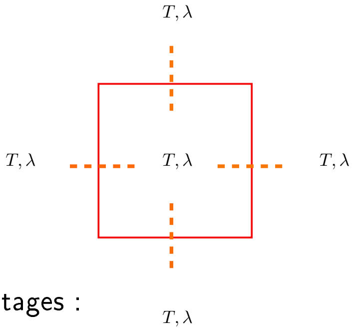




Tomography, J. Escoda, 2011

Studing diffusion by finite volumes on a regular grid



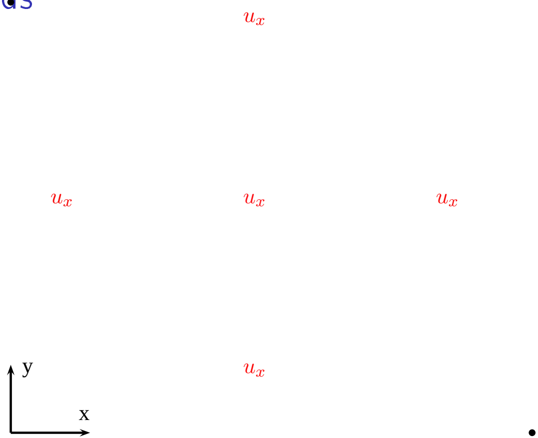
The local behaviour is isotropic

Advantages :

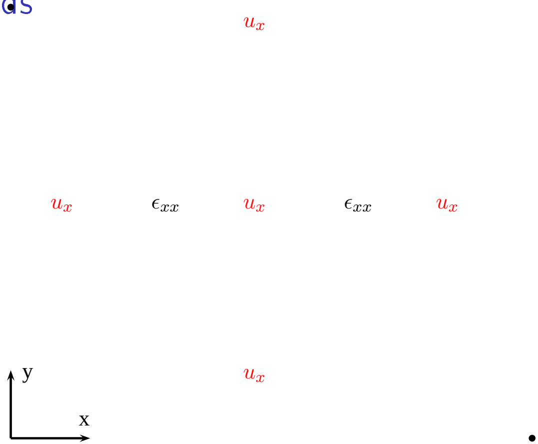
- precise at order 2 in space within the domain, concervative
- The matrix is small : sparse, symmetric $\Longrightarrow$ 4 non-nul terms per line.
- The matrix is positive semi-definite $\Longrightarrow$ iterative solvers such as preconditionned conjugate gradient are available.(lower RAM consumption, good scalability)

$\Longrightarrow$ Large problems can be treated

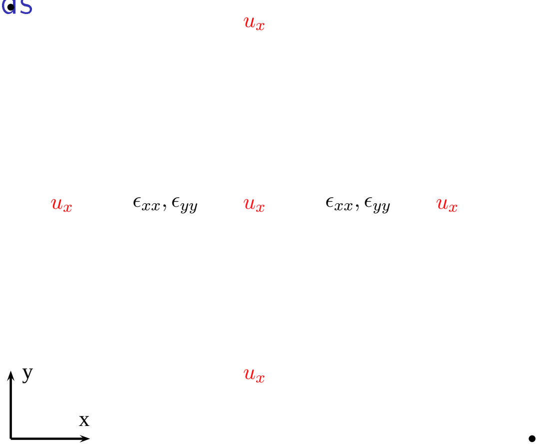
Studing elasticity by centered finite differences on regular grids



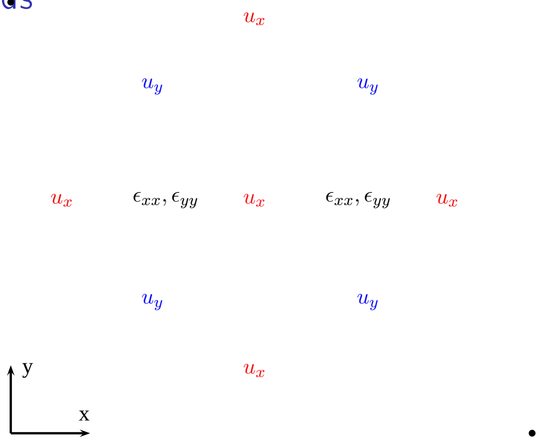
Studing elasticity by centered finite differences on regular grids



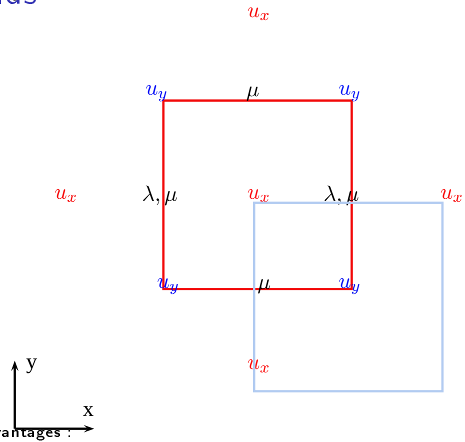
Studing elasticity by centered finite differences on regular grids



Studing elasticity by centered finite differences on regular grids



Studing elasticity by centered finite differences on regular grids



The local behaviour is isotropic, compressible.

Seismic Waves : J. Virieux 1984

Elasticity : Y. Zhu, E. Sifakis, J. Teran, A. Brandt 2010

Advantages :

- precise at order 2 in space within the domain, concervative
- The matrix is small : sparse, symmetric
  $\Longrightarrow$ 8 non-nul terms per line (in fact 7 3x3 blocs per 3 lines)
- The matrix is positive semi-definite

$\Longrightarrow$ Large problems can be treated.

- The implementation is based on PETSC
  Portable Extensive Toolkit for Scientific Computation
- Algorithm : full multigrid V-cycles.
- Aim : an optimal complexity $O(N)$

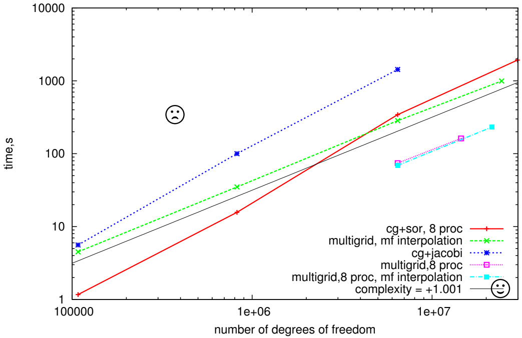


Figure: B11, elasticity, displacement BC, $E_i/E_m = 100$, $\nu = 0.2$, on HP Z600 12Go RAM, $21x10^6 dofs, 233s$

Scalability : Speed Up

$$Speed\_Up(nb_{proc}) = \frac{t_{sequential}}{t_{parallel}(nb_{proc})}$$

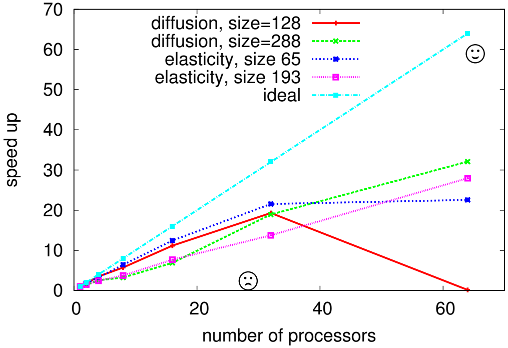


Figure: B11, diffusion with periodic BC and elasticity with displacement BC, multigrid algorithms

Big cases are more scalable than small ones.

Scalability : scale up, toward bigger cases

Thanks to distributed memory and a large memory band,
it is possible to work on large cases within an acceptable time.

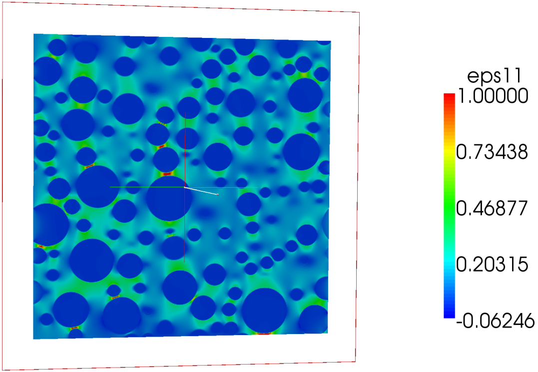


Figure: B11, elasticity, displacement BC, $E_i/E_m = 100$, $size = 513$, $dofs = 4x10^8$, t=65s with 64 nodes, 8ppn on cluster Clamart2

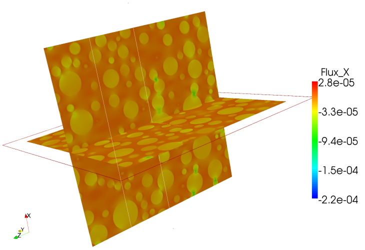


Figure: B11, diffusion, periodic BC, $\lambda_i/\lambda_m = 100$, $size = 512$, $dofs = 1.3x10^8$, t=25s with 32 nodes

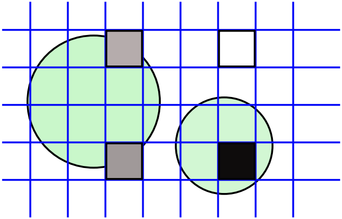
How to ensure reliability ?

- Compare results with analytical ones.
  Homogeneous material, laminar composites.
- Compare with fields produced by the finite element method
- Benchmark of many codes. Exemple on B11, diffusion, λ_i/λ_m

λ_i/λ_m,	$\lambda_{FDVGRID}$	λ_{FFT} *	HS +or-
100, grey=Reuss	$3.258\lambda_m$	$3.242\lambda_m$	$2.918\lambda_m$
0.01, grey=Voigt	$0.4915\lambda_m$	$0.4942\lambda_m$	$0.504\lambda_m$

* : The implementation of the FFT algorithm is the one of François Willot, Centre de Morphologie Mathématique, Mines Paristech

Effects of grey elements



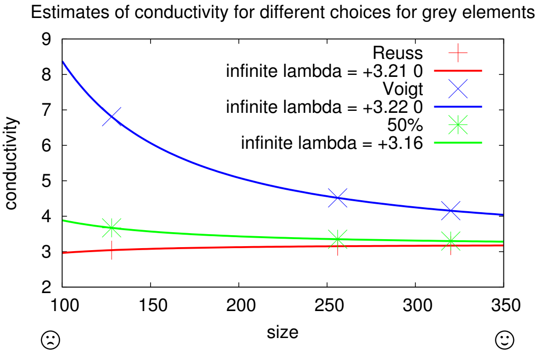


Figure: B11, diffusion, $\lambda_i/\lambda_m = 100$, periodic BC

As the resolution increases, different rules for grey elements lead to closer estimates of behaviour.

Effects of boundary conditions

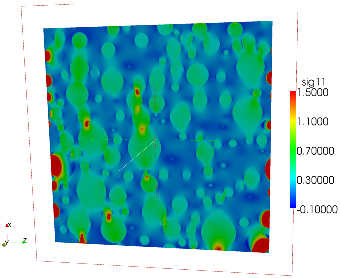


Figure: B11, elasticity, displacement BC, $E_i/E_m = 100$

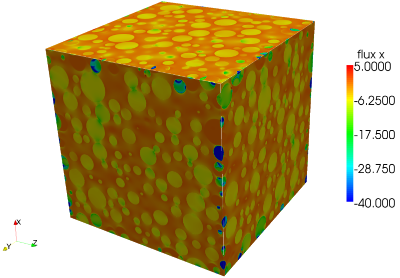


Figure: B11, diffusion, periodic BC, $\lambda_i/\lambda_m = 100$

$\Longrightarrow$ Boundary effects hinder the precision of the estimated behaviour.
The contrast ($E_{inclusions}/E_{matrice}$ here) between phases matters a lot.
Low contrast $\Longrightarrow$ good estimated behaviour

No boundary effects in the case of periodic microstructures with periodic boundary conditions.

The Material Ageing Plateform

The Material Ageing Plateform MAP gathers many codes of MMC

- Chaining codes easily
- Documentation and usability
- A common build
- Non-regression tests

components : c_solver_diffusion_fdvgrid and c_solver_elasticity_fdvgrid
scheme : s_concrete_st1 : elastic homogeneization in concrete

Conclusion

c_solver_diffusion_fdvgrid :

- isotropic diffusion, Fick law / Fourier law
- Dirichlet, Neumann, periodic and Newton boundary conditions
- Source term

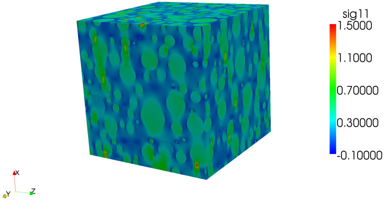
c_solver_elasticity_fdvgrid :

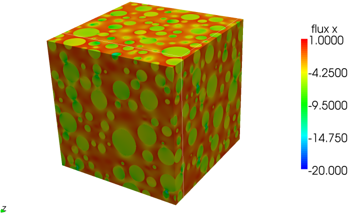
- isotropic elasticity
- periodic, displacement and force boundary conditions
- Porosity, Prestress

Towards long term viscoelasticity :

- Using Laplace-Carson transform ⇒ high contrast between phases ($E_{inclusions}/E_{matrice} > 100$)
- Or time stepping ⇒ it requires fast computation
  Implicit formulation : an elastic problem at each step.

Questions ?





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