INVERSE ANALYSIS TAILORED FOR BOTH STRAIN HARDENING AND STRAIN SOFTENING UHPFRC

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Abstract

In this article, a computational time-efficient model predicting bending behavior which surpasses simplified models and gives the results with the similar accuracy as the fiber-beam models is presented. The time efficiency is due to the fact that the proposed model works in the moment-curvature space directly, thus avoiding the computational costs related to section integration which has to be performed with the fiber-beam models. The model is based on a modified force-based fiber-beam formulation where progressive loading is driven by a curvature at its non-linear hinge. The curvature outside of the nonlinear hinge decreases during a deflection-softening phase and therefore a damaged constitutive law should be introduced. Rather than using the classical damage model proposed by Mazars, a macroscopic damage model at the moment-curvature level is proposed. The moment-curvature damage model reduces the whole computation of the beam equilibrium to only one numerical loop. Hypotheses of the new model such as damage modeling, localization, and shear deflection are briefly discussed and the model's practical applications are presented to show the benefits.

Résumé

Dans cet article, on présente un modèle économe en temps de calcul pour prédire le comportement en flexion qui surpasse les modèles simplifiés et donne des résultats d'une précision semblable aux modèles de poutres multifibres. Le gain de temps est dû au fait que le modèle proposé travaille directement dans l'espace moment-courbure, évitant ainsi les coûts de calcul liés à l'intégration de la section qui doit être réalisée avec les modèles multifibres. Le modèle est basé sur une formulation multifibres modifiée, exprimée en force, où le chargement progressif est piloté par la courbure au niveau de la rotule plastique. La courbure à l'extérieur de la rotule non linéaire diminue pendant la phase de comportement adoucissant et, par conséquent, une loi de comportement avec endommagement doit être introduite. Plutôt que d'utiliser le modèle d'endommagement classique proposé par Mazars, un modèle d'endommagement macroscopique au niveau moment-courbure est proposé. Le modèle d'endommagement moment-courbure réduit la totalité du calcul de l'équilibre de la poutre à une seule boucle numérique. Les hypothèses du nouveau modèle telles que la modélisation de l'endommagement, la localisation et la déformation de cisaillement sont brièvement discutées et les applications pratiques du modèle sont présentées pour en montrer les avantages.

1. INTRODUCTION

Identification of non-linear properties of ordinary concrete and especially ultra-high performance fiber reinforced concrete (UHPFRC) is a subject of many research projects. Uni-axial tensile and compressive tests are the most appropriate methods as they directly measure the desired properties. However, the uni-axial tensile test is complicated to perform as well as to analyze due to a localization phenomenon. Two kinds of non-linear descriptions are used because of the localization: strain and crack-opening displacement (COD). Whilst the strain description is a natural continuation of elastic properties, the COD description can be only used after initiation of a macroscopic crack. On the other hand, the COD can be measured easily with a notched specimen tested in uni-axial tension whereas the strain measurement has to be deduced from the measured displacement by assuming a characteristic length. Due to a lack of sufficient information on the characteristic length and complexity of the uni-axial tensile tests, the complete strain description is usually deduced from indirect measurements. The indirect measurements are also preferable for the COD because the overall configuration is significantly simpler than the uni-axial tensile tests.

Three-point and four-point bending tests are recommended for UHPFRC [1, 2]. However, the indirect measurement of standard or arbitrary bending configurations requires additional treatment of the experimental results to evaluate the exact tensile properties. The additional treatment, also called back-analysis, is divided into three groups: a standard method used inversely, a simplified method used inversely and a straightforward inverse method.

The first possible method for the post-treatment of the experimental results is the standard method used inversely. It deduces the material properties from an iterative procedure where the desired properties are firstly estimated and then iteratively corrected. The standard method used inversely simulates the experimental results with models based on finite elements (fig.1e) or fiber-beam elements (fig.1d). Although the finite element models represent a very powerful tool for versatile modeling, they also represent the most time-consuming approach. Since many UHPFRC applications can be simplified into less complex structures, the fiber-beam element models are more appropriate. A comprehensive study on the fiber-beam element models can be found in Valipour and Foster [2]. They summarized all the important milestones and models in this field since the very beginning. Force-based or mixed-based formulations are generally considered more suitable and they were used in much research. Although the force-based or the mixed-based fiber-beam element models are generally faster than the finite element models, they are still time-consuming for simply-supported structures. Therefore, simplified methods used inversely and straightforward inverse methods are more popular due to their higher time efficiency.

The simplified method used inversely employs the same approach as the standard methods used inversely. However, due to its simplicity, each iterative loop is shorter, and consequently, the total computation time is significantly reduced. The simplest model lumps the entire nonlinear deformation into one hinge and the rest is considered to behave as a rigid body (fig.1a). The complete force-deflection relationship with a post-peak section can be derived because the entire nonlinearity is lumped into one nonlinear hinge. The replacement of the rigid body by a pure elastic behavior improves predictive capacities (fig.1a). An additional improvement is obtained by adding a shear contribution to the total deflection which then provides good results for materials with deflection-softening behavior. The previous models are no longer valid for materials with deflection-hardening behavior because the assumption of the rigid or linear body underestimates the total deflection. Chanvillard and Corvez [4] proposed a new model to avoid the underestimation of deflection which takes into account the nonlinear behavior of the entire body (fig.1b) with an arbitrary deflection-hardening material. This model uses the force-based approach and explicitly evaluates the force-deflection relationship up to the peak load.

The straightforward inverse method [1, 5, 6, 7] directly connects the experimental results in bending to the uni-axial constitutive law (stress-strain or stress-COD relationships). The straightforward inverse methods differ in complexity: certain methods can evaluate an arbitrary constitutive law [7] while others only evaluate a predefined constitutive law. The main benefit of the straightforward inverse method is its straightforwardness which eliminates any iterative loops (estimation and correction). Nevertheless, these methods are mainly tailored for back-analyzing of material properties and can be hardly used for precise predictions of various responses.

In this paper, a precise method (fig.1c) that can back-analyze/simulate various responses and governs all the models in the simplified method group (fig.1a & b) is presented. The method provides results with the similar accuracy as the fiber-beam models but is undeniably time efficient compared to the standard methods. The proposed model uses the theory of fiber-beam elements with a modified force-based formulation. The modified force-based (M–FB) formulation can overcome difficulties with a deflection-softening material (localization) and it also includes deflection-hardening material. The M–FB model is designed for statically determinate structures. The model also takes into account a shear contribution to the total deflection. Due to the framework of the fiber-beam elements, the M–FB model has a versatile range of use and it can also analyze concrete beams with rebars. The M–FB model is optimized with respect to its computational time efficiency, on account of being used inversely or for reliability-based code calibrations based on Monte Carlo simulations.

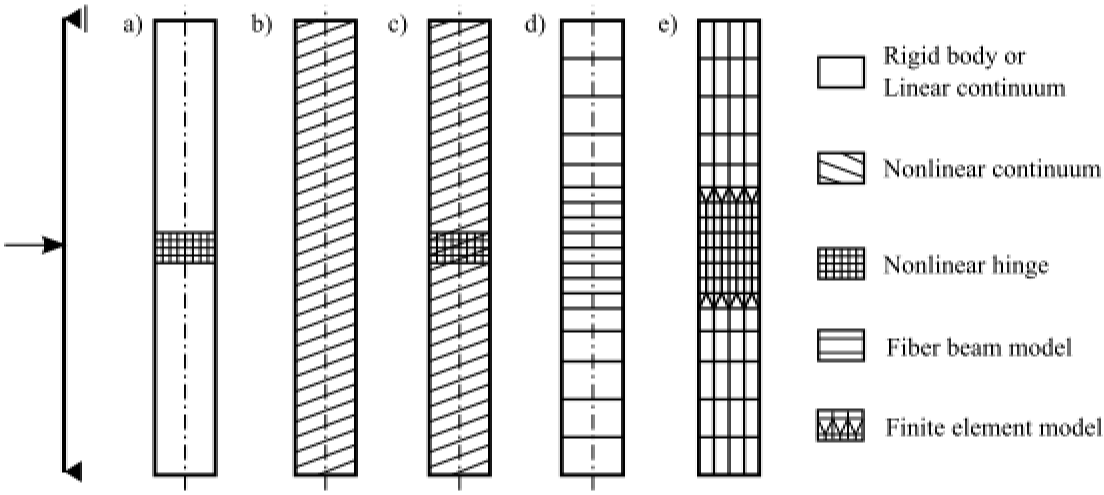


Figure 1: Various complexity of numerical models

2. MODEL DESCRIPTION

In statically determinate structures, the internal forces are known and consequently, the force-based formulation is ideal for such structures. The main shortcoming of the force-based formulation is its inability to describe the behavior after the onset of the softening, unless special and hence time-consuming approaches are used.

Figure 2 (Beam section) illustrates the proposed model where the modified force-based formulation is used. The progressive loading is driven by the curvature of the non-linear hinge instead of the force. This technique has two main advantages; the progressive loading driven by curvature helps overcome the peak load and continues on the softening section; it can also capture a snap-back effect in the force-displacement relationship. The complete model consists of four steps: a new curvature of the non-linear hinge is established for a given stress-strain constitutive law (1); the cross-section equilibrium is solved and the corresponding moment in the nonlinear hinge is computed (2); once the moment in the nonlinear hinge is known, the classic force-based formulation is used for the known distribution of the moment of the rest of the beam (3-4); the deflection of the beam is derived from the curvature, the moment and the shear force distributions.

The notable benefit of the proposed model comes from simplification of the structural equilibrium. The complete process to determine the state of the structure for one loading step consists of only one iterative loop at the cross-section level for an arbitrary combination of material within the cross-section.

2.1 Notation

The actual loading step is denoted by superscript "t" (X^t), the previous loading steps are denoted by superscript "τ" (X^τ), the nonlinear hinge element is denoted by subscript "cr" (X_{cr}), position of the elements along the span for the moment and the curvature is denoted by subscript "j" (X_j) and the position of the fiber along the height of the cross-section is denoted by subscript "k" (X_k),

2.2 Cross-section equilibrium

The cross section equilibrium in two dimensions is written as,

$$\int_A \sigma_x dA - N_x = 0 \tag{1}$$

$$\int_A \sigma_x \left(y - y_g\right) dA - M_z = 0 \tag{2}$$

where y and y_g are the distances from the top surface to the integration point (layer) and to the neutral axis, respectively, σ_x is the stress at the integration point (layer), N_x is the axial force and M_z is the bending moment.

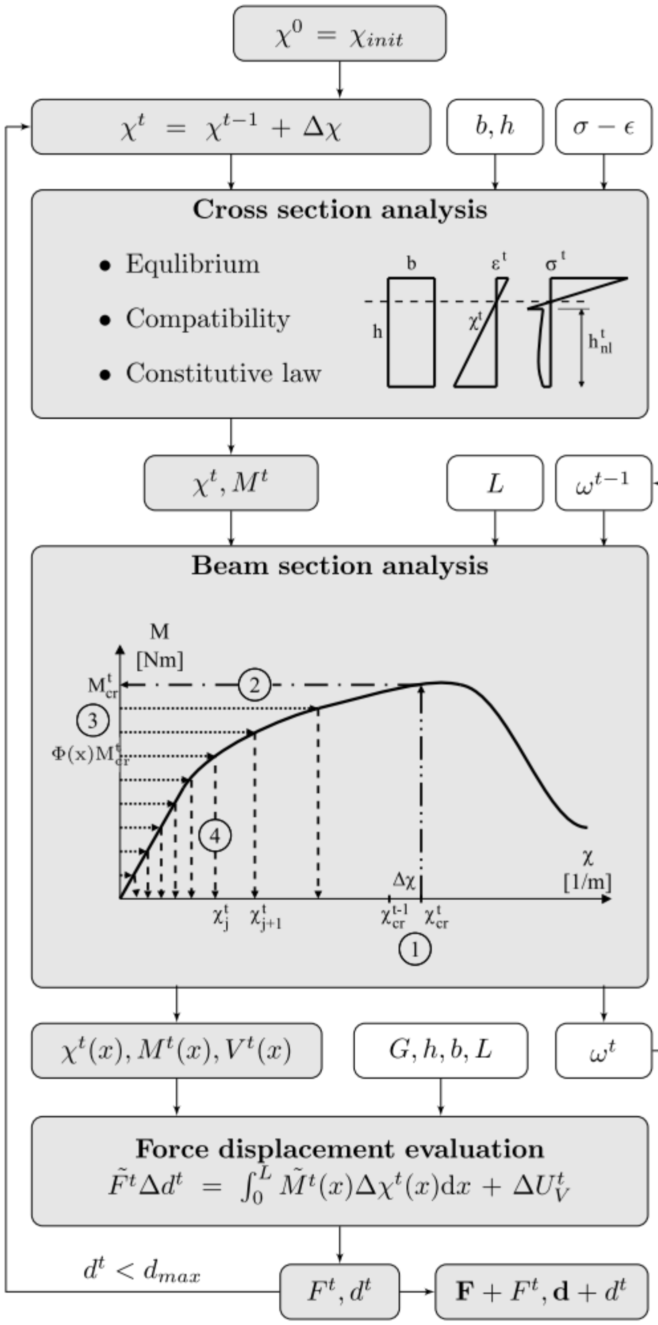
The compatibility requirement, for the Navier-Bernoulli theory without a bond slip, is

$$\varepsilon_x = \varepsilon_r + y\chi \tag{3}$$

where ε_x denotes the axial strain at the integration point (layer), ε_r denotes the axial strain at the top surface and χ denotes the curvature of the cross-section. The uni-axial stress-strain relationship of the nonlinear material is represented by,

$$\sigma_x = f\left(\varepsilon_x\right) \tag{4}$$

A combination of previous equations creates an iterative loop to obtain the position of the neutral axis (y_g) for the given axial force (N_x). Once the position of the neutral axis is known, the corresponding moment (M_z) is computed.



1. Starting with an initial curvature

2. Updating the curvature

3. Computing the moment-curvature relationship at the cross-section level

4. Applying material damage

5. Computing moment and curvature distributions

1 The new curvature step

2 Cross-section equilibrium for evaluation of a corresponding moment at the nonlinear hinge

3 Moment distribution derived from a moment interpolation function

4 Curvature distribution derived from the moment-curvature relationship considering damage theory

6. Updating damage parameters

7. Deriving deflection from the curvature, moment and shear force distributions (or as a double integration of the curvature distribution considering a shear effect)

8. Storing the results

Figure 2: Modified force-based formulation algorithm

2.3 Structural equilibrium

Each loading step is described by the curvature ($\chi_{cr}^t = \chi_{cr}^{t-1} + \Delta\chi$) and the corresponding moment (M_{cr}^t) at the nonlinear hinge. Considering $\Phi(x)$ as a moment interpolation function along the span, the moment distribution is,

$$M^t(x) = \Phi(x) M_{cr}^t \tag{5}$$

A link (Â) between the moment distribution and the curvature distribution is a linear interpolation between the closest points. The corresponding couple (χ_j^t, M_j^t) is searched in the moment-curvature relationship derived for the nonlinear hinge (fig.2 – Beam section).

The curvature outside the nonlinear hinge decreases when the softening behavior occurs. To satisfy the cross-section equilibrium with partially damaged layers of material, a damaged unloading path must be used. In the M-FB model, the damage model [8] is applied to the moment-curvature relationship (fig.3b). This simplifies the nonlinear computation of the cross-section equilibrium into one formula,

$$\chi_j^t = \min\left(\frac{M_j^t}{\left(1-\omega_j^t\right)k_e}, \hat{A}\left(M^t(x)\right)\right) \tag{6}$$

where ω_j^t is the damage variable and k_e is the elastic flexural stiffness. The damage parameter is updated every load step to fulfill the following condition,

$$\left(1-\omega_j^t\right)k_e = \min_{\tau<t}\left(\frac{M_j(\tau)}{\chi_j(\tau)}\right) \tag{7}$$

2.4 Force-deflection relationship

Deflection at each loading step is derived in an incremental form from a principle of energy conservation as proposed by Timoshenko [9].

$$\widetilde{F}^t \Delta d^t = \int_0^L \widetilde{M}^t(x)\Delta x^t(x)dx + \Delta U_V^t \tag{8}$$

where F is the applied load, d is the deflection under the applied load, L is the length of the beam and ΔU^t_V is the energy associated with the shear forces along the length of the beam. Alternatively, the force-deflection relationship can be derived from a double integration of the curvature distribution.

3 DISCUSSION

In the previous chapter, the proposed model was described without deeper discussion of two topics that are important for the proposed model. In this chapter, both topics are discussed in depth: a simplification of the structural equilibrium (Moment-curvature damage model) and a fiber beam discretization with the localization issue.

3.1 Damage model applied to the moment-curvature relationship

The simplest version of continuum damage mechanics is a uni-axial damage model. In the standard fiber beam model, two unique damage parameters are used for each fiber and then the cross-section equilibrium is computed. If the cross-section is composed of materials all obeying the Mazars' damage law, and unloading occurs, the axial cross-section equilibrium can be written in the following discrete form,

$$N^t = \sum_{k=0}^{m} E_k\left(1-\omega_k^{t-1}\right)\left(\chi^{t-1}+\Delta\chi^t\right)\left(y_k - \left(y_g^{t-1}+\Delta y_g^t\right)\right) \tag{9}$$

where m is the total amount of fibers at the cross-section, $E_k(1-\omega_k^{t-1})$ is the apparent stiffness of the material, $\Delta\chi$ is the unknown unloading step and Δy_g is the corresponding change of the neutral axis. Eq.9 is derived with the assumption that each layer undergoes unloading. Therefore, the damage parameters do not change during the step ($\omega_k^t = \omega_k^{t-1}$). Due to a constant axial force, a difference between the previous and actual loading steps is equal to zero. With the axial force equal to zero (the majority of experimental set up), $\sum E_k(1-\omega_k^{t-1})(y_k-y_g^{t-1})$ is zero. Consequently, Δy_g must be zero as well. Hence, the moment resistance is simplified into a new form from which the change of the curvature can be derived as:

$$\Delta\chi_j^t = \frac{\Delta M_j^t}{M_j^{t-1}}\chi_j^{t-1} \tag{10}$$

Eq.10 provides the same curvature increment as eq.6 during unloading, which verifies the moment-curvature damage model. Fig.3 compares the standard stress-strain damage model and the moment-curvature damage model. Both loading paths are identical because the cross-sections are composed of materials that all obey Mazars' damage law.

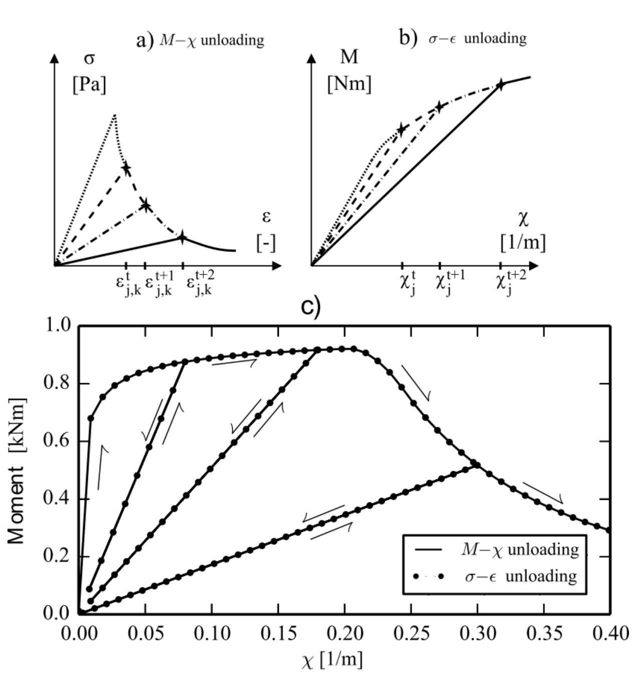


Figure 3: Damage models for concrete: the standard stress-strain damage law (a), the moment-curvature damage law (b); difference between them for UHPFRC [12] (c)

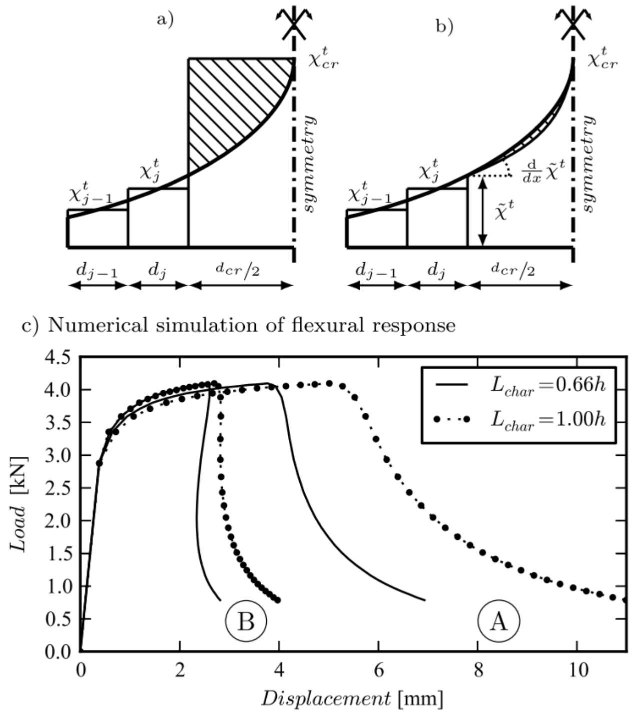


Figure 4: Error of numerical integration on the non-linear hinge: rectangular rule "A" (a), parabolic shape "B" (b), sensitivity to the size of the element of "A" and "B" (c)

3.2 Fiber beam discretization and localization problem

Eq.8 is straightforward but also problematic because the localization occurs at the onset of the deflection softening. Due to a longitudinal discretization higher number of elements (integration points) causes a shorter length of elements. Consequently, the global response has spurious mesh sensitivity. In the proposed model, an idea from the crack-band theory [10] is used, where the localization is lumped into one crack band with a constant effective element size d_{cr} equal, in our case, to the characteristic flexural length l_{fl}. Elements outside the crack band (nonlinear hinge) do not localize which is why, the size of the elements can be arbitrary. The length of the crack band (nonlinear hinge - l_{fl}) d_{cr} is one of the input parameters.

The rectangular rule for numerical integration outside the nonlinear hinge is sufficiently precise due to the monotonically-increasing curvature over the length of the beam. However, fig.4 shows errors if the rectangular rule is applied on the nonlinear hinge. Another drawback of the rectangular rule for the nonlinear hinge is its direct proportionality to the size of the element. The length of the nonlinear hinge is generally empirically derived; hence it does not have a unique value. Different standards and recommendations lead to a strong sensitivity to the size of the element. In the proposed model, these two drawbacks are minimized assuming the symmetrical parabolic shape of the curvature within the nonlinear hinge as proposed by Casanova [11]. Fig.4 shows differences between the rectangular rule and the assumed parabolic shape for ordinary concrete with two different characteristic flexural lengths.

4. DATA ANALYSIS

Two possible applications of the proposed model are presented in this section: a fast inverse analysis of unknown material and a parametric study of facade panels made of a concrete matrix, organic fibers, and nonmetallic reinforcement. Another possible application of the model is related to Monte Carlo simulations and reliability-based code calibrations presented as a separated paper in this proceeding.

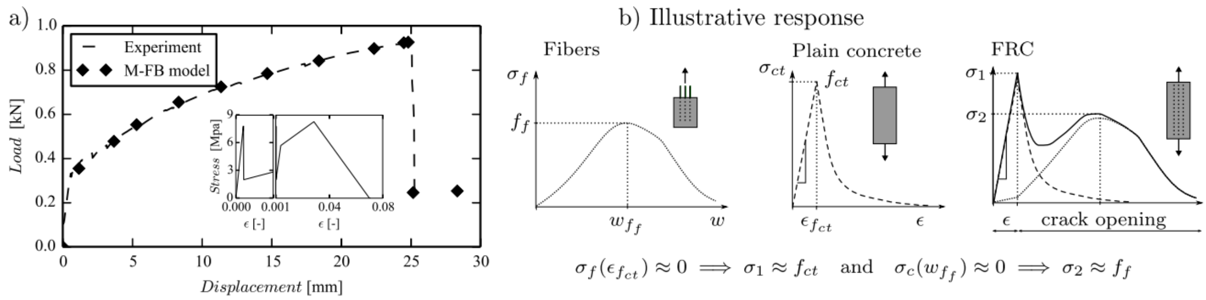


Figure 5: (a) Inversely-analyzed stress-strain relationship (small figure) from the experimental results of the unknown plate, (b) simplified superposition of fibers and matrix contribution to the response of UHPFRC

4.1 Inverse analysis

Identification of nonlinear concrete parameters represents an important role in material research. A task of this study was to identify nonlinear properties of a material from which a set of plates (500/150/13mm) were produced. The proposed model was used to inversely analyze the experimental results obtained by 3-points bending tests. The tests were performed at LafargeHolcim R&D center according to the UHPFRC recommendations [1] where the piston displacement was continuously increased until the complete failure. The plate displacement was measured at the middle of the span by linear variable displacement transducers (LVDT). After the experimental testing, the input properties of the material were randomly generated and the numerical simulations and the experimental results were compared on the basis of the root-mean-square error. The entire process was automated by a computer script using Simulating Annealing and it successfully converged to the “real” unique material parameters without any additional intervention over a very short period of time (a few minutes for the whole convergence and a few seconds for one force-deflection simulation). Fig.5a shows a good capacity of the proposed model to capture the real response. The inversely-analyzed stress-strain relationship had an expected shape for UHPFRC as it is shown on fig.5b. The tensile resistance of the matrix is relatively low (compared to other UHPC matrices) but the high volume of fibers ($\sigma_2 > \sigma_1$) helps to increase the overall resistance of the tested plates.

4.2 Parametric study

The parametric study focused on identification of a contribution of different materials in a composite panel. The studied panels (500/150/15mm) were composed of a UHPC matrix, organic fibers and two layers of a symmetrical non-metallic reinforcing mesh placed 5mm from each side. Firstly, the complete composite panels (concrete matrix, fibers, and reinforcement) were tested in four-point bending. The tests were conducted at LafargeHolcim R&D center according to the UHPFRC recommendations [1] where the piston displacement was continuously increased until the complete failure. The panel deflection was measured at the middle of the span by LVDTs. After the experimental testing, the complete composite panel was inversely-analyzed with the proposed model. Fig.6a shows the experimental results and the fitted numerical equivalent of the complete composite panel. Once the properties of each material were known (inversely-analyzed), some components of the composite panel (fibers or reinforcing mesh) were removed to numerically simulate various responses. After the numerical simulations, auxiliary experimental tests, with the identical testing conditions, were carried out to confirm the simulations (matrix-fibers and matrix-mesh). Fig.6b shows very good conformity between the experimental results and the numerical simulations. The study proved a good capacity of the proposed model to simulate different behaviors without additional experimental testing.

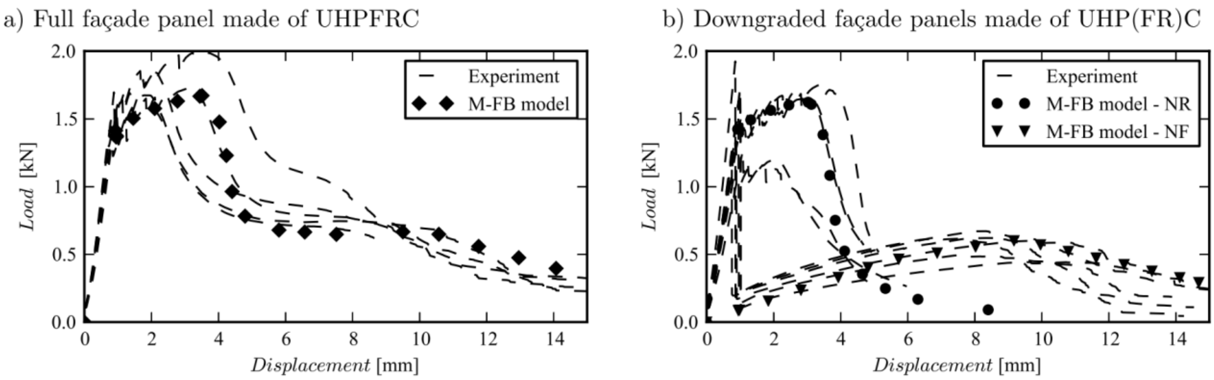


Figure 6: (a) Numerical simulation of facade panels made of UHPFRC with non-metallic reinforcement, (b) numerical and experimental parametric study with facade panels made of UHPC, NR: UHPFRC without reinforcing mesh, NF: UHPC with reinforcing mesh

5. CONCLUSION

Prior work documented the effectiveness of the force based fiber-beam model for a broad range of concrete structures. Nevertheless, the force-based fiber-beam model is still superfluously robust and time-consuming for most experimental set ups, although it is generally faster than the finite element models.

Simplified models are, therefore, often used to avoid time inefficiency. However, none of the simplified models can compete with the standard models, namely for the deflection-hardening materials, even though the models differ in their complexity. The main shortcomings of the simplified models are: the deflection-hardening behavior, the numerical treatment of the localization and the shear contribution.

The proposed model surpasses all the simplified models and gives the results with the similar accuracy as the fiber-beam models (standard methods) with undeniable computational time efficiency. The time efficiency is due to the fact that the proposed model works in the moment-curvature space directly, thus avoiding the computational costs related to section integration which has to be performed with the fiber-beam models. The modified force-based formulation driven by its curvature overcomes difficulties for the deflection-hardening behavior and the subsequent deflection-softening behavior. Moreover, it can capture a snap-back effect directly without any additional treatment. Possible errors from the numerical integration are minimized with the assumed shape of the curvature in the nonlinear hinge.

This paper showed the very good conformity between the numerical simulations and the experimental results (fig.6 and 7). The computational time efficiency and the accuracy provide a broad range of applications for experimental and statistical work.

In addition to the mentioned findings, the moment-curvature damage law was verified and its suitability for UHPFRC was discussed. The moment-curvature damage law is equivalent to the standard damage law proposed by Mazars [8] for the fiber-beam element models. This finding may significantly accelerate standard fiber-beam element models.

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