PROBABILISTIC NUMERICAL MODEL OF ULTRA-HIGH-PERFORMANCE FIBRE REINFORCED CONCRETE (UHPFRC) CRACKING PROCESS

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Abstract

The objective of this paper is to demonstrate the validity of a probabilistic explicit cracking model developed for current SFRCs to simulate the behaviour of an Ultra-High Performance Fibre Reinforced Concrete (UHPFRC) beam. The parameters characterizing the tensile behaviour of the studied UHPFRC are determined from the 3-point bending tests performed on beams sawed from the reinforced UHPFRC beam. The results suggest that the probabilistic explicit cracking model is relevant to analyze the mechanical behaviour of an UHPFRC beam and provide precise information about the cracking process of this type of material. The good agreement between the experimental result and the numerical simulations permits to conclude that the studied UHPFRC behaves like current FRC, it means that it can be modelled using the same type of cracking model.

Résumé

Cet article a pour objectif de démontrer qu'un modèle probabiliste de fissuration explicite développé pour analyser les bétons de fibres courants reste valide pour simuler le comportement d'une poutre en Béton Fibré à Ultra-hautes Performances (BFUP). Les paramètres caractérisant le comportement en traction du BFUP étudié sont déterminés à partir d'essais de flexion 3-points. Les résultats obtenus dans le cadre de cet article permettent de conclure que le modèle probabiliste de fissuration explicite est en effet pertinent pour analyser la fissuration des structures en BFUP (informations précises sur l'ouverture et l'espacement des fissures). Ils permettent, également, de confirmer le fait que le BFUP étudié dans le cadre de cet article se comporte mécaniquement comme un béton de fibres courant.

1. INTRODUCTION

New construction methods and technologies are in constant evolution in order to expand the lifespan of existing and new structures. As part of the new technologies, new advancements in concrete and cement-based products are gradually changing the design and construction worlds. Materials such as ultra-high performance fibre reinforced concrete (UHPFRC) are increasingly used in structural applications. One of the main reasons of its popularity is related to the existence of national (French (AFGC-SETRA [1]), German (DAfStb [2]), Japanese (JSCE [3]), Italian (CNR-DT 204 [4])) and international (FIB [5]) recommendations for the design of UHPFRC structures.

These recommendations are efficient and conservative for designing simple statically determinate structures subjected in bending. However, they do not possess a sufficient physical base to propose relevant solutions for more complex structures such as statically indeterminate structures or more complex loadings as shear or punching situations. Moreover, the control of cracking in the serviceability limit state is one of the main interests of using UHPFRC. Crack control is very important for durability aspects and is certainly a major advantage of UHPFRC in comparison to structures using traditional reinforcement bars. Nowadays, existing design recommendations do not provide sufficient relevant information regarding cracking at the serviceability limit state (crack opening and spacing). In this way, the best approach for designing structures with respect to both safety and sustainable development is the use of finite element analysis.

The objective of this paper is to prove the validity of the probabilistic explicit cracking model developed for current SFRCs to simulate the behaviour of UHPFRC beam and, by this way, to demonstrate that UHPFRCs having a compressive strength not exceeding 300 MPa and a percentage of fibres not exceeding 3% (in volume) show a cracking process and a mechanical behaviour in tension comparable to the ones observed in more classical SFRC [6].

The probabilistic explicit cracking model has been in constant development since the late 1980 [7-13]. An improvement of this probabilistic explicit cracking model to permit the analysis of steel fibre reinforced concretes (SFRC) cracking was proposed and validated for different type of SFRC with compressive strengths not exceeding 90 MPa [10-13].

2. PROBABILISTIC EXPLICIT CRACKING MODEL OF STEEL FIBRE REINFORCED CONCRETE (SFRC)

The model is based on three main physical evidences that have been observed experimentally:

- Concrete is a heterogeneous material in which the heterogeneities can be modeled through a random spatial distribution of mechanical property considered as dominant in the cracking process, namely the tensile strength [7].
- Scale effects of concrete cracking are taken into account by the fact that mechanical properties of the material such as the tensile strength depend on the size of the mesh elements chosen for the finite element analysis [7, 14]. In contrast, the average post-cracking energy is considered independent of scale effects [15].
- Cracking is explicitly treated through the creation of random kinematic discontinuities, which provides access to quantitative information on the cracking state (number of cracks, opening and spacing). Numerically speaking, these cracks are represented by interface elements.

The crack initiation criterion may be summarized as following: the interface element opens when the normal tensile stress at the centre of the interface element reaches a critical value. The critical value corresponds to the tensile strength of the material probabilized through a Weibull distribution function [9]. This critical value depends on the total volume of the two volumetric elements interfaced by the considered interface element. This means that the stress and the rigidity of the interface element briefly become equal to zero until the bridging effect of the fibres takes over. The criterion of crack creation is schematically presented in Figure 1.

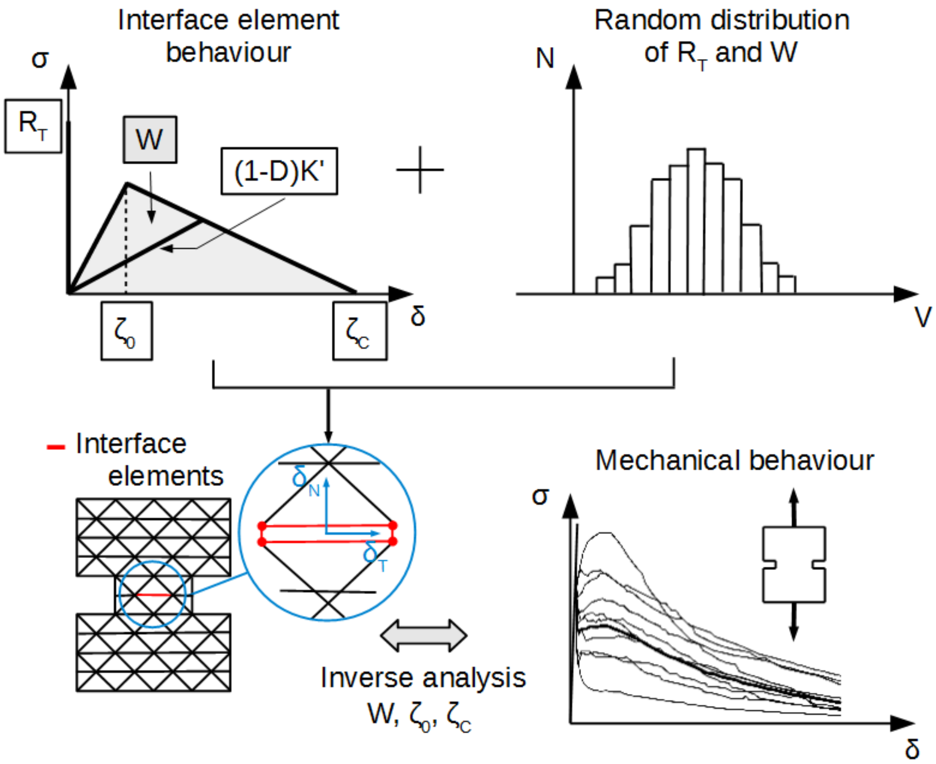


Figure 1: Principles of the explicit probabilistic cracking model of SFRC

Therefore, the creation of cracks in the cement matrix is represented by an elastic perfectly brittle behaviour, whereas the bridging effect of the fibres is described by the following modelling approach.

Normal and tangential stresses in the interface element linearly increase with normal and tangential displacements when a "broken" interface element re-opens to take into account the elastic effect of the fibres inside the crack. Physically speaking, the rigidity of the fibres (inside the cracks) is more important in tension than in shear. Thus, the interface element rigidity is considered different for normal and tangential displacements. In 2D, normal and tangential rigidities of the interface element are K_n' and K_t' respectively. The post-cracking elastic behaviour exists until it reaches a threshold value, ζ_0, related to the normal displacement (Figure 1). The mechanical behaviour of the interface element changes once this threshold value is reached. The normal stress is considered as linearly decreasing with the normal displacement in order to take into account the damage of the bond between the concrete and the fibre, and fibre pullout. The decreasing evolution is obtained by using a damage model.

Finally, the interface element is considered definitively broken when the normal displacement reaches a threshold value, ζ_c (Figure 1). This value corresponds to the state where the effect of fibres is considered negligible. It is determined from a uniaxial tensile test. At this point, its normal and tangential rigidities are set to zero.

The post-cracking energy dissipated by the bridging effect of the fibres is considered randomly distributed on the mesh elements. The random distribution chosen is a log-normal distribution function with a mean value independent of the mesh elements size [15] and a standard deviation, due to the heterogeneity of the material, increasing as the mesh elements size decreases. The choice of a log-normal distribution function is an arbitrary one. It is convenient to avoid having negative values of the post-cracking energy when the element meshes are very small.

The threshold parameters ζ_0 and ζ_c are determined by an inverse analysis approach to best fit the simplified triangular stress-displacement curve representing the post-cracking energy (Figure 1) to the experimental tensile softening curve.

In order to consider a softer tangential behaviour, the tangential stiffness, K_t', is taken as proportional to the normal stiffness considering a coefficient α. A value of α = 0.3 was determined from shear tests on SFRC beams [11].

3. NUMERICAL MODELLING OF A REINFORCED UHPFRC BEAM SUBMITTED TO SHEAR LOADING

The probabilistic explicit cracking model developed for SFRC has been used to simulate the mechanical behaviour and cracking process of a longitudinally reinforced UHPFRC beam without transverse reinforcement bars. The beam was tested in an experimental campaign performed on the tensile behaviour of UHPFRC at the structural level [16].

3.1 Experimental Details

The beam was designed to develop a shear failure mechanism. Therefore, its longitudinal reinforcement was designed to avoid bending failure. The beam's cross section and its reinforcement and the test set-up are respectively presented in Figure 2. The reinforcement consisted of 5-HA20 and 1-HA25, the latter being placed in the middle of the bottom layer of rebars. The test as a 4-point bending test, consisting of three roller supports and one pinned. The spans between the loading points and the supports were respectively 480 and 2000 mm.

The UHPFRC mix design used was a self-compacting concrete made of Portland cement, fine sand with a maximum aggregate size of 0.8mm, silica fume, high-range water-reducing admixture, and straight steel fibres with a length of 13 mm and a diameter of 0.2 mm at 2.5 % by volume. The UHPFRC had a 28-day mean compressive strength of 212 MPa, a Young's modulus of 56 GPa and a Poisson's ratio of 0.185.

To evaluate the influence of fibre orientation on the tensile behaviour of the studied UHPFRC, two series of small beams were tested under 3-point bending. The two series were sawn from the reinforced UHPFRC beam after the test. The first series was made of beams sawn at a 45° angle to the reinforced UHPFRC beam length and the second series was made of beams sawn in parallel to the reinforced UHPFRC beam length. These small beams were 280 mm long, 70 mm high and 60 mm wide, with a 2 mm wide and 8 mm deep notch sawn at mid-span from a single blade stroke. The support span was set to 210 mm. It should be noted that the small beams dimensions were not exactly identical as the beams were sawn from a larger beam. Therefore, the aforementioned dimensions represent the average dimensions of the small beams.

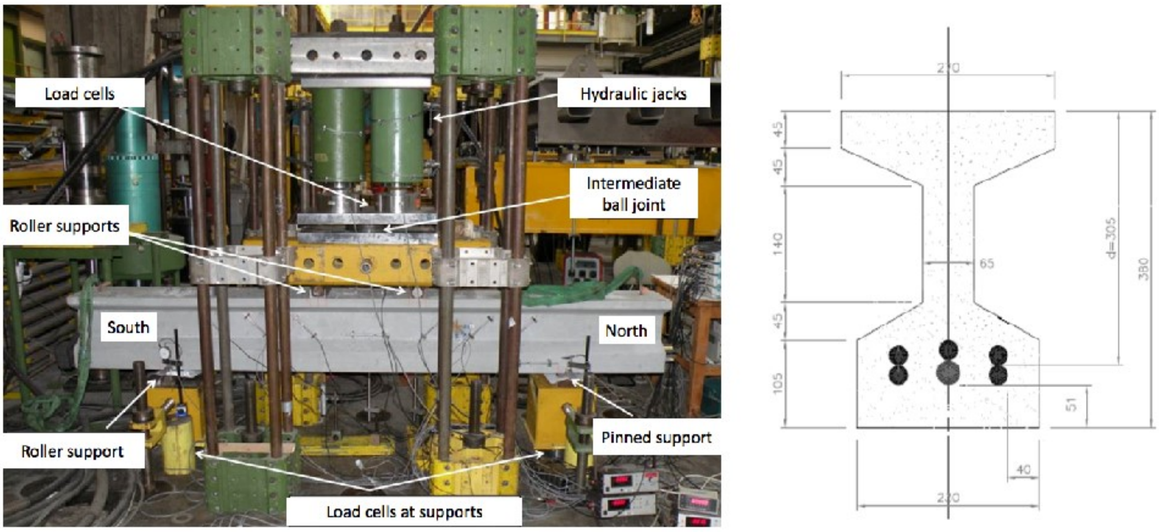


Figure 2: Cross section of the reinforced UHPFRC beam and experimental testing device

3.2 Characterization of the tensile behaviour of UHPFRC

Although some general parameters such as the 28-day compressive strength, Young's modulus and Poisson's ratio have been directly determined experimentally, the mechanical characteristics of the tensile behaviour of the studied UHPFRC must be determined by inverse analysis. The inverse analysis approach consists of simulating the 3-point bending tests performed on the small notched beams sawn from the reinforced UHPFRC beam to identify the average tensile strength (R_T) and its standard deviation ($\sigma(R_T)$), the average post-cracking energy (W) and its standard deviation ($\sigma(W)$), and the threshold parameters ζ_0 and ζ_c defining the simplified triangular representing the post-cracking energy.

Generally, the tensile strength and its scattering can be determined using the analytical relations presented in [8]. These analytical functions correspond to the distribution function of the tensile strength with regards to scale effects. Therefore, they mainly depend on the volume of the mesh elements and the maximum aggregate size. In fact, the maximum aggregate size characterizes the heterogeneity of the material, which is in turn at the origin of the scaling effect of the tensile strength of the rupture in tension. However, these analytical equations were obtained for concretes having compressive strengths between 30 and 130 MPa, in which the maximum aggregate size was much larger than the fibres. In contrast, the high volume of fibres combined with their dimensions (compared with the very small diameter of the aggregates used) make the fibres the main source of heterogeneity in the UHPFRCC studied. They induce local stresses concentrations around them and facilitate the initiation of microcracks in the matrix and result in a reduction of the tensile strength. Therefore, the mean tensile strength and its scattering are determined by best fitting the linear elastic segment of the load-deflection curves obtained from the 3-point bending tests.

Figure 3 presents the finite element mesh used to model the 3-point bending tests to identify the mechanical characteristics of the tensile behaviour by inverse analysis. Ten two-dimensional simulations have been performed in plane stresses conditions.

Figure 4 presents the numerical load-crack opening curves obtained from the 3-point bending tests on the notched beams sawed respectively at a 45° angle and parallel (at 0°) to the UHPFRC beam length in comparison to the experimental results

The results show a good agreement between the experimental results and the numerical simulations for both orientations of sawed beams.

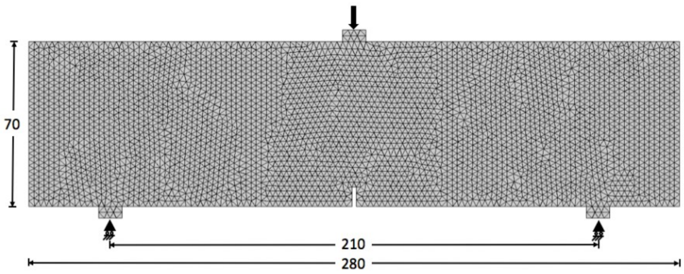
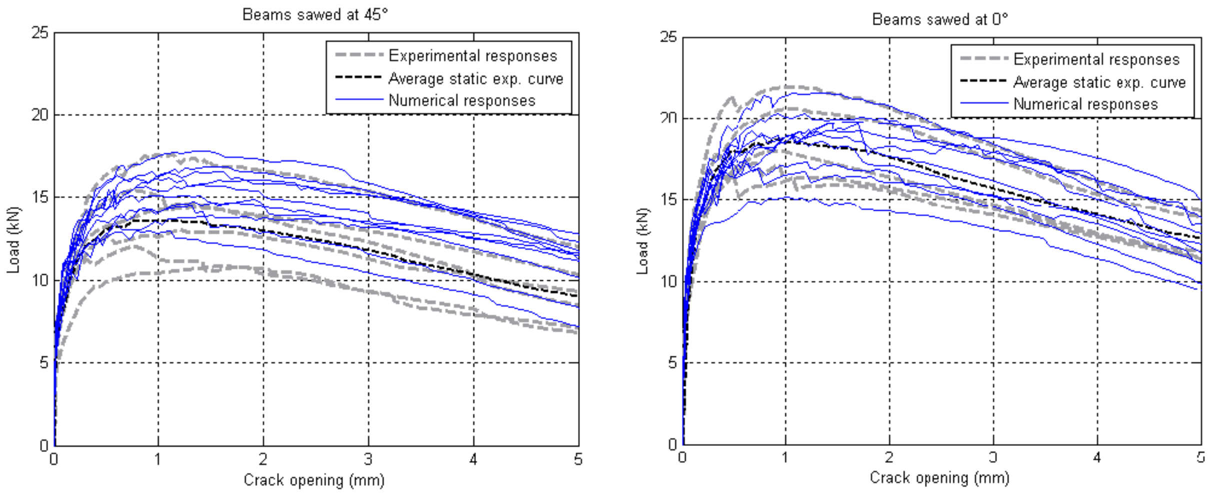


Figure 3: Finite element mesh of the of notched beams tested in 3-point bending



(a) (b)

Figure 4: Experimental and numerical load-crack opening curves of the beams sawed at 45°(a) and sawed at 0°(b)

As expected, the beams sawed parallel to the length of the beams present a greater peak load and post-cracking energy than the beams sawed at 45° due to the orientation of the fibres. Indeed, fibres are expected to be oriented parallel to the length of the beam as a result of the casting process, more precisely the flow of the self-compacting UHPFRC mix. In addition, Figure 4 shows that the initial loss of rigidity of the beams sewed at 45° and at 0° correspond to loads of approximately 7 and 10 kN respectively . This may be explained by the fact that well oriented fibres sew cracks more effectively.

The model parameters obtained through the inverse analysis approach are summarized in Table 1. In addition, the parameters ζ_0 and ζ_c that best fitted the experimental results obtained from the 3-point bending tests are 0.035 mm and 5 mm.

Table 1: Values of the material parameters used in the numerical model

	f_t [MPa]	$\sigma(f_t)$ [MPa]	W [MPa*mm]	σ(W) [MPa*mm]
Beams sawed at 45°	8.43	1.31	60	72
Beams sawed at 0°	9.46	0.48	70	84

3.3 Numerical Simulation of the Shear Behaviour of the Beam

In reinforced concrete beams subjected to shear forces, shear cracks form diagonally, hence at 45° from the length of the beam. The shear failure mechanism is characterized by the presence of diagonal shear cracks either before or after the formation of flexural cracks. In that respect, the set of parameters that best represent the orientation of the fibres with regards to diagonal cracks corresponds to the parameters identified on the characterization beams sawed at 45°.

Figure 5 presents the finite element mesh used for the reinforced UHPFRC beam simulations.

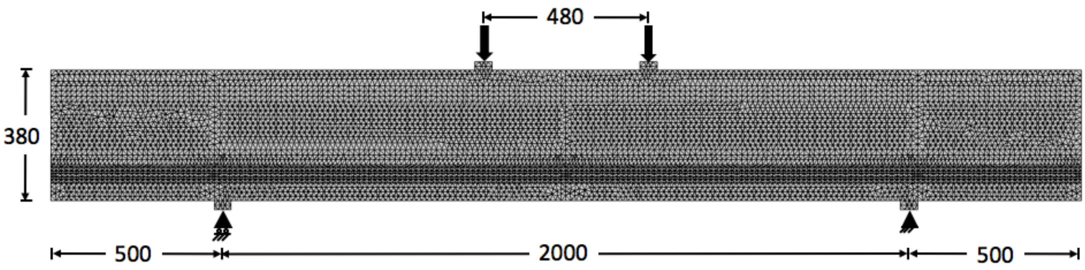


Figure 5: Finite element mesh of the reinforced UHPFRC beam

The fineness of the mesh has been chosen to be similar to the one chosen for the small specimen loaded in bending (Figure 3). So, in the spirit of the proposed approach, the set of parameters given in Table 1 can be used for the present simulations. For other meshes fineness, the set of parameters should be different. So, it can be considered that the delicate point of the proposed modelling approach is linked to the determination of this set of parameters.

Three two-dimensional simulations have been performed in plane stresses conditions. The reinforcement bars were modelled with linear continuum elements having a height corresponding to the rebar's diameter. Before cracking, the special linear interface elements ensure the continuity of stresses and displacements by using a very high matrix of rigidity. In fact, they are similar to those used for simulating cracking of SFRC without rebars. After cracking, the rebar's effect on the cracks, thus on the interface elements, is considered as a linear elastic behaviour. This behaviour is modelled by setting the normal and tangential components of an interface element's stiffness to be much smaller than before cracking [11]. The values of these stiffness components are determined through inverse analysis by best fitting the elastic rigidity of the beam (corresponding to the bending cracks created first).

Figure 6 presents a comparison between the experimental load-deflection curve and the three numerical simulations.

The results show a good coherence between the experimental result and the numerical simulations until a deflection of 5 mm, after which the numerical simulations underestimate the experimental behaviour. This may be explained by the numerical identification method for the tensile behaviour of the studied UHPFRC. As mentioned previously, the tensile behaviour of the studied UHPFRC was identified by an inverse analysis approach that consisted on simulating the 3-point bending tests performed on the notched beams sawed from the reinforced UHPFRC beam. Experimentally, it has been observed that multiple cracks initiate at the tip of the notch, therefore not representing the tensile behaviour of the material but rather a structural behaviour in which multiple cracks are sewed by a group of fibres. Therefore, this indirect approach generally underestimates the tensile behaviour of the material due to the stress concentrations occurring at the tip of the notch.

Figure 7 presents an example of a cracking pattern obtained from the numerical simulations at a deflection of 5 mm. Only cracks larger than 100 μm are shown on the figure. The crack pattern shows that several bending cracks were created at the bottom fibre of the beam. Although bending cracks are first created, their propagation is rapidly blocked by the bending reinforcement. These cracks are mainly responsible for the initial loss of flexural rigidity observed at approximately 180 kN, after which the behaviour of the beam is controlled by its shear behaviour. The crack pattern shows that three diagonal cracks with a maximum crack opening of 0.25 mm have formed. Their propagation is controlled by the bridging effect of the fibres as the beam has no stirrups.

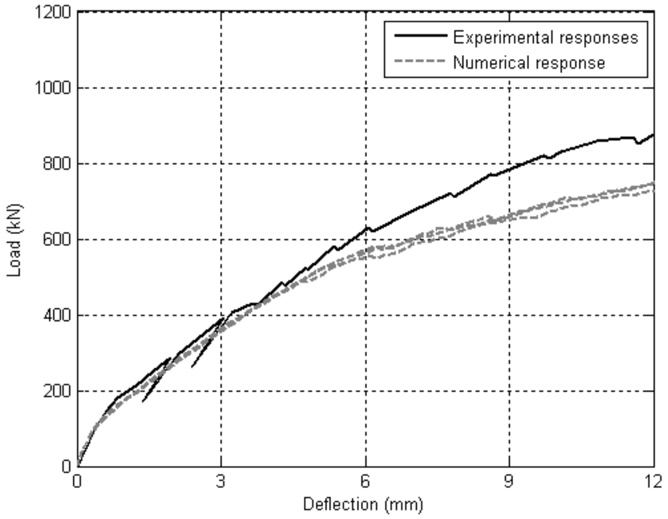


Figure 6: Experimental and numerical load-deflection curves of the reinforced UHPFRC beam submitted to shear loading

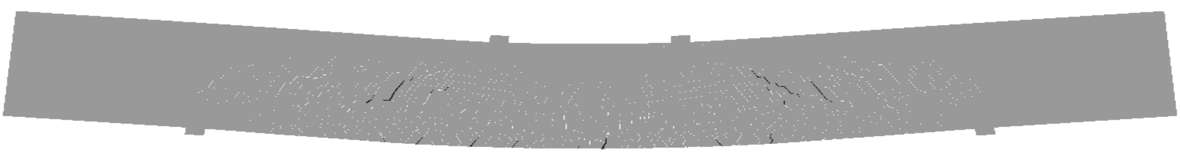


Figure 7: Example of cracking process of the reinforced beam loaded in shear

Figure 8 presents a comparison between the experimental load-deflection curve and the three numerical simulations with the same set of parameters but a 33% increase of the post-cracking energy distribution. It means that this mean post-cracking energy and its standard deviation were respectively increase from 60 to 80 MPa*mm and from 72 to 96 MPa*mm. The results show that the shear behaviour is comparable to the experimental behaviour up to a deflection of 6 mm, after which the model underestimates the shear behaviour of the beam by less than 10%. This confirms that the 3-point bending tests underestimate the post-cracking energy of the tensile behaviour.

To summarize, the results show a good agreement between the experimental result and the numerical simulations both at the global and local scales. Therefore, the results suggest that the studied UHPFRC behaves like usual FRC as the tensile strength and the mechanical effect of the fibres may be modelled by a perfectly brittle behaviour and a nonlinear softening behaviour.

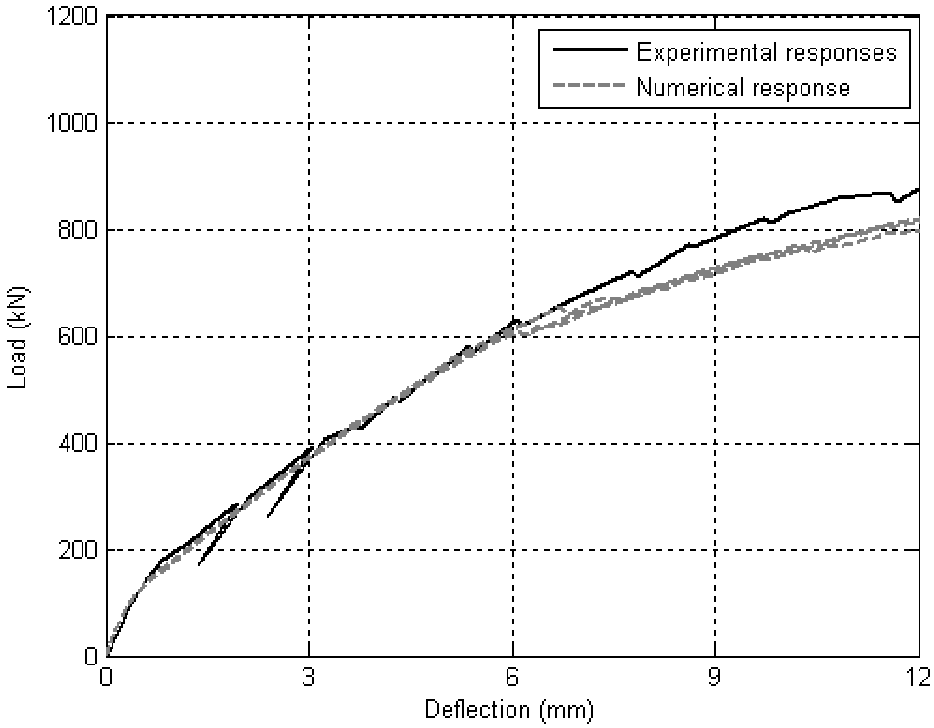


Figure 8: Experimental and numerical load-deflection curves of the reinforced UHPFRC beam submitted to shear loading with increased post-cracking energy

4. CONCLUSIONS

The objective of this paper is to verify the validity of the probabilistic explicit cracking model developed for SFRC to simulate the behaviour of a reinforced UHPFRC beam subjected to a bending load leading to shear failure. The relevancy of the model is evaluated through the simulation of the mechanical behaviour of the beam. The parameters characterizing the tensile behaviour are determined from the 3-point bending tests performed on beams sawed from the reinforced UHPFRC beam.

The main conclusion of this work can be summarized as follows.

- The probabilistic explicit cracking model is relevant to analyze the mechanical behaviour of a reinforced UHPFRC beam and provide precise information about the cracking process of this type of material.
- The good agreement between the experimental result and the numerical simulations suggest that the studied UHPFRC (2.5 % by volume of fibres and a compressive strength of 212 MPa) behaves like usual FRC as the tensile strength and the mechanical effect of the fibres may be modelled by a perfectly brittle behaviour and a nonlinear softening behaviour.

REFERENCES.

[1] AFGC-SETRA (2013). 'Ultra High Performance Fibre-Reinforced'. *A. Publications*. France
[2] DAfStb (2007). 'Guidelines for Steel Fibre Concrete' – *23 th Draft – Richtlinie Stahlfaserbeton – DIN 1045. Annex parts 1-4, Deutcher Ausschuss für Stahlbeton*.
[3] JSCE (2008). 'Recommendations for Design and Construction of High Performance Fibre Reinforced Cement Composites with Multiple Fine Cracks (HPFRCC)', *Japan Society of Civil Engineers (JSCE)*.
[4] CNR-DT 204 (2006). 'Guidelines for Design, Construction and Production Control of fibre reinforced concrete structures', *National Research Concil of Italy*.
[5] FIB (2013). 'fib Model Code for Concrete Structures 2010'. Lausanne, *The International Federation for Structural Concrete (fib)***:** 402.
[6] Rossi, P. (2008). 'Ultra-high performance fibre reinforced concretes (UHPFRCC): A summary of current knowledge.' *Concrete International* **30**(2): 31-34.
[7] Rossi, P. and S. Richer (1987). 'Numerical Modelling of Concrete Cracking Based on a Stochastic Approach.' *Materials and Structures* **20**(119): 334-337.
[8] Rossi, P. and X. Wu (1992). 'Probabilistic model for material behaviour analysis and appraisement of concrete.' *Magazine of Concrete Research* **44**(161): 271-280.
[9] Tailhan, J.-L., et al. (2010). 'From Local to Global Probabilistic Modeling of Concrete Cracking.' *Annals of Solid and Structural Mechanics* **1**: 103-115.
[10] Rossi, P., et al. (2014). 'Numerical modelling of cracking in Steel Fibre Reinforced Concretes (SFRC) structures.' *Euro-C 2014 : Computational Modelling of Concrete Structures*. H. M. Nenad Bicanic, Gunther Meschke, René de Borst. St. Anton am Arlberg, CRC Press. **2:** 463-471.
[11] Tailhan, J.-L., et al. (2015). 'Probabilistic numerical modelling of cracking in steel fibre reinforced concretes (SFRC) structures.' *Cement and Concrete Composites* **55**: 315-321.
[12] Rossi, P., et al. (2015). 'Analysis of Cracking in Steel Fibre Reinforced Concrete (SFRC) Structures in Bending using Probabilistic Modelling.' *Structural Concrete, Journal of the fib*.
[13] Rossi, P., et al. (2016). 'Numerical models for designing steel fiber reinforced concrete structures: why and which ones ?' *FRC 2014: ACI-fib International Workshop, Editors: B. Massicotte, J.-P. Charron, G. Plizzari, B. Mobasher, FIB Bulletin 79 – ACI SP-310*, 289-300.
[14] Rossi, P., et al. (1994). 'Scale effect on concrete in tension.' *Materials and Structures* **27**: 437-444.
[15] Rossi, P. (2012). 'Experimental Study of Scaling Effect Related to Post-Cracking Behaviours of Metal Fibres Reinforced (MFRC).' *European Journal of Environmental and Civil Engineering* **16**(10): 1261-1268.
[16] Baby, F. (2012). 'Contribution à l'identification et la prise en compte du comportement en traction des BFUP à l'échelle de la structure.' *PhD thesis,Université Paris-Est*, 522 (in French).