Variable Crack-Angle Softened-Truss Model for Shear in Reinforced Concrete Beams

J S KUANG
Associate Professor
Hong Kong University of Science & Technology
Hong Kong

Pak Cheung CHENG
PhD Candidate
Hong Kong University of Science & Technology
Hong Kong

Summary

This paper presents a new and rational theory for predicting the shear behaviour of reinforced concrete membrane elements and beams subjected to combined shear and axial load, the Variable Crack Angle Softened Truss Model (VCA-STM), which is used to the entire load-deformation history including the slip deformation across the crack and obtain the change of the crack angle of concrete, which is non-coincident to the directions of the principal stress and strain. This model can reasonably describe the appearance of secondary cracks for members with less transverse reinforcement and predict the shear stress transmitted by friction at the cracks. Test of 37 specimens are used to verify the calculated shear strength by the proposed model and is shown to be in good agreement.

Keywords: axial loads; beams; crack angle; Mohr circle; reinforced concrete; shear strength; tension.

1. Introduction

Large-scale structures such as shear wall, bridges, containment structures and offshore structures…etc are required to design with significant longitudinal reinforcement, but the traditional shear design rules were developed from tests of members, which were not reinforced for combined shear and axial load. The typical and rational models such as Modified Compression Field Theory (MCFT) [1, 2], Rotating Angle Softened Truss Model (RA-STM) [3] and Fixed Angle Softened Truss Model (FA-STM) [4, 5] were commonly applied for predicting the nonlinear behaviour of cracked reinforced concrete membrane elements. However, there are so-called "conceptual errors" of the MCFT and RA-STM have been pointed out by some researchers [6, 7]. The MCFT and RA-STM assume the crack angle of concrete coincides with the direction of principal compressive stress of concrete and this causes a problem that the so-called "contribution of concrete" would vanish and cannot be predicted [7, 8]. The FA-STM assumes the angle of cracks is fixed and equal the angle of principal compressive stress with respect to longitudinal steel bars, usually 45 degree, this cannot reflect the change of crack angle for members with less or no stirrups as test observations showed that while failure occurred, there were the appearance of secondary cracks 9). In this paper, the variable crack angle softened truss model (VCA-STM) is proposed to predict the shear stress-strain relationships of reinforced concrete membrane elements including the slip deformation across the crack. This model assumes the angle of cracks of concrete is changing from initial crack up to failure by considering the cracks exist at the current stage for a given value of load and allows the angles of cracks is non-coincident with the directions of the principal stress and strain of concrete.

2. Variable Crack-Angle Softened-Truss Model

2.1. Stress and strain formulations

The applied stresses (f_x, f_y and v_{xy}) acting on the element with concrete principal directions in the 2,1 coordinate are represented in Fig. 1. This model assumes that the angle of cracks, α in the concrete is non-coincident with the principal angle of cracked concrete, θ.

The equations for average stresses and strains of cracked concrete in the variable crack angle softened truss model are expressed as follows

$$f_{cx} = f_{c1} - v_{cxy} \cot\theta \tag{1}$$

$$f_{cy} = f_{c1} - v_{cxy} \tan\theta \tag{2}$$

$$v_{cxy} = \left(\frac{f_{c1} - f_{c2}}{2}\right) \sin 2\theta \tag{3}$$

$$\varepsilon_{cx} = \left(\frac{\varepsilon_{c1} - \varepsilon_{c2}}{2}\right)(1 - \cos 2\theta) + \varepsilon_{c2} \tag{4}$$

$$\varepsilon_{cy} = \varepsilon_{cx} + (\varepsilon_{c1} - \varepsilon_{c2}) \cos 2\theta \tag{5}$$

$$\gamma_{cxy} = 2(\varepsilon_{cy} - \varepsilon_{c2}) \tan\theta \tag{6}$$

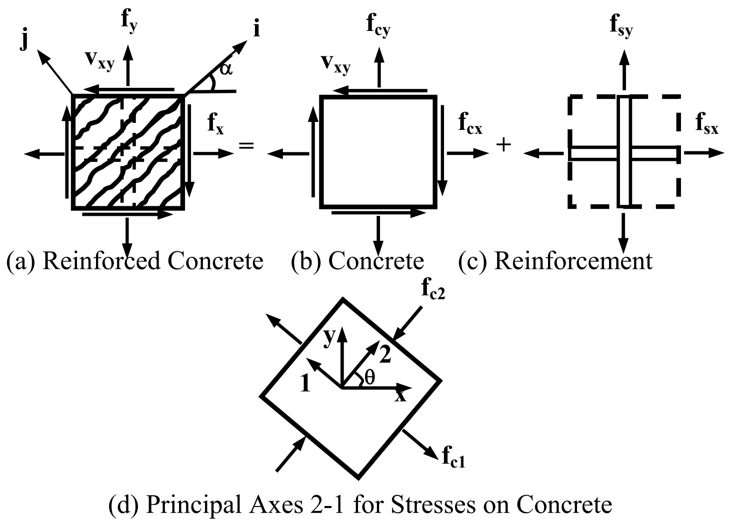


Fig. 1 Stresses acting on reinforced concrete membrane elements

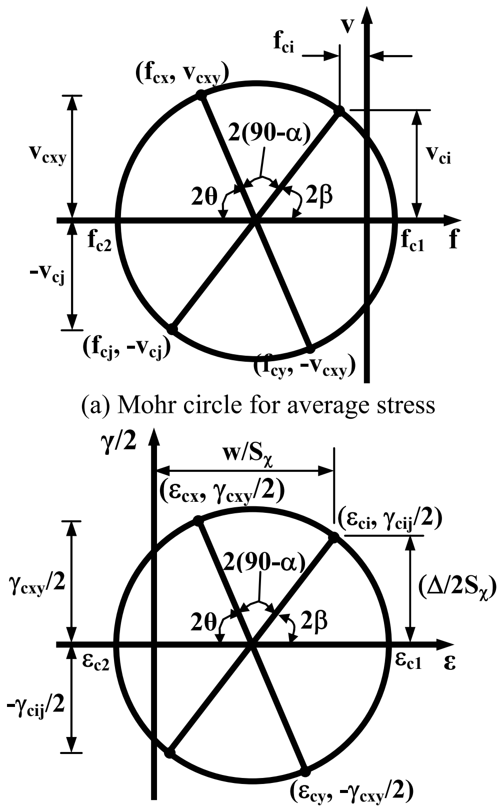
where f_{cx}, f_{cy} are the average concrete stresses in x- and y- directions respectively, v_{cxy} is the average shear stress of concrete in the x, y coordinate, ε_{cx}, ε_{cy} are the average concrete strains in x- and y- directions respectively, γ_{cxy} is the average shear strain of concrete in the x, y coordinate, f_{c1}, f_{c2} are the average principal stress of concrete in 2,1 directions respectively, and ε_{c1}, ε_{c2} are the average principal strain of concrete in 2,1 directions respectively.

In the VCA-STM, the angle of cracks of concrete, α is changing for the loading process, it can be derived by the using the stresses and strains relationship on the cracked surface from the Mohr circle as shown in Fig. 2.

$$v_{ci} = \frac{f_{c1} - f_{c2}}{2} \tan 2(\alpha - \theta) \tag{7}$$

Since the main inputs for generating the entire shear stress-strain response are ε_{c1}, ε_{c2}, θ and α (which will be discussed in 2.4), the above equation includes two unknowns, which are the principal inclination of concrete and angle of cracks, θ and α respectively. In order to determine the crack angle, α, one more equation is required for solving the concrete shear stress, v_{ci}, which will shown in 2.3.

2.2. Constitutive laws



(a) Mohr circle for average stress

(b) Mohr circle for average strain

Fig. 2 Mohr circle for cracked concrete

The principal compressive and tensile stresses, f_{c2} and f_{c1} can be obtained by the following constitutive laws:

$$f_{c2} = \zeta f_c' \left[2\left(\frac{\varepsilon_{c2}}{\zeta \varepsilon'_c}\right) - \left(\frac{\varepsilon_{c2}}{\zeta \varepsilon'_c}\right)^2 \right] \text{ (Compression)} \qquad (8)$$

where f_c' is the maximum cylinder strength of concrete (MPa), ε'_c is the concrete strain at peak stress and ζ is the softening coefficient equals $\left(5.8/\sqrt{f_c'}\right)\cdot\left(1/\sqrt{1+400\varepsilon_{c1}}\right)$.

If $\varepsilon_{c1} < 0.00008$, $\quad f_{c1} = E_c \varepsilon_{c1}$ (9)

If $\varepsilon_{c1} \geq 0.00008$, $\quad f_{c1} = f_{cr}\left(0.00008/\varepsilon_1\right)^{0.4}$ (10)

where f_{cr} is the cracking stress of concrete equals $0.31\sqrt{f_c'(\text{MPa})}$ and E_c is the elastic modulus of concrete taken as $3875\sqrt{f_c'(\text{MPa})}$.

The stresses acted on the longitudinal reinforcement and transverse reinforcement can be calculated by using the following constitutive laws:

If $\varepsilon_s < \varepsilon_{np}$, $\quad f_s = \varepsilon_s E_s$ (11)

If $\varepsilon_s \geq \varepsilon_{np}$, $f_s = f_{yd}\left[\left(0.91 - 2B\right) + \left(0.02 + 0.25B\right)\cdot\left(\varepsilon_s/\varepsilon_y\right)\right]$ (12)

where ε_{np} is the average steel tensile strain at first yield defined as $\varepsilon_{yd}(0.93 - 2B)$ and B is the parameter equal to $(1/\rho)\cdot(f_{cr}/f_y)^{1.5}$ for $\rho \geq 0.5$ %.

2.3. Concrete shear stress

In the VCA-STM, the angle of cracks, α is a input parameter and keep changing for the entire load-deformation history and can be determined by equation (7). The shear stress-strain relationship of concrete can be required to solve the angle of cracks, α, the expression of equation of concrete in shear is proposed by Pang and Hsu (1996) [4]:

$$v_{ci} = v_{ci\max}\left[1 - \left(1 - \frac{\gamma_{cij}}{\gamma_{cij0}}\right)^6\right] \qquad (13)$$

where γ_{cij0} is the maximum concrete shear strain, taken as $-0.04\varepsilon_{c10}(\alpha - \theta)$, ε_{c10} is the maximum principal tensile strain of cracked concrete, and v_{cimax} is the maximum shear stress of concrete, defined as $v_{ci\max} = \left[\left(f_x - \rho_{sx} f_{sx}\right) - \left(f_y - \rho_{sy} f_{sy}\right)\right]\left(\sin 2\alpha\right)/2 + v_{xy\max}\cos 2\alpha$, where,

$v_{xy\max} = \dfrac{\left(v_{ci\max}\right)^2}{2\sqrt{\rho_{sx} f_{sxyd}'' \rho_{sy} f_{syyd}''}} + \sqrt{\rho_{sx} f_{sxyd}'' \rho_{sy} f_{syyd}''}$ and f_{syd}'' is the local yield stress at cracks defined as $f_{yd}\left[1 - \left(2 - \alpha/45\deg\right)/1000\rho\right]$.

2.4. Solution scheme

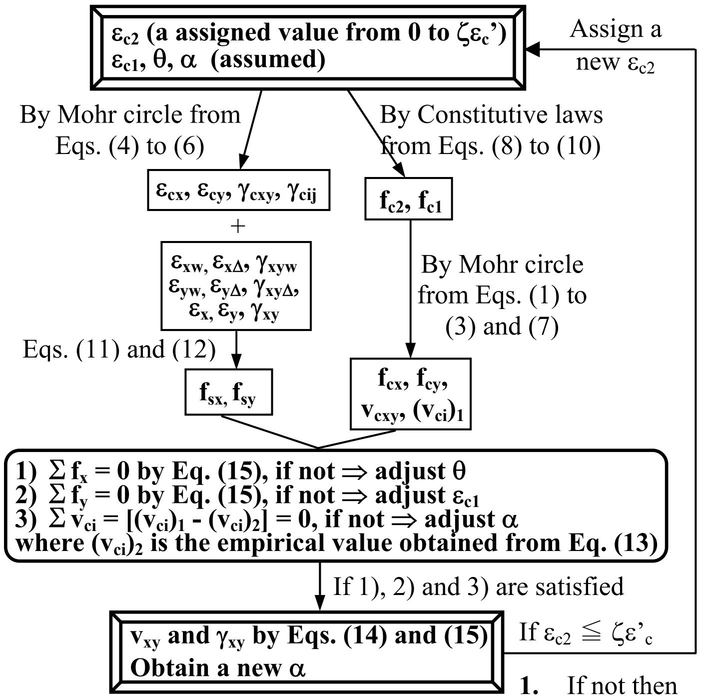


Fig. 3 Flow Chart

The total strains are required for determining the steel stresses and checking the stress equilibrium of the reinforced concrete membrane elements, which can be expressed by summing up the average and local strains.

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \varepsilon_{cx} \\ \varepsilon_{cy} \\ \gamma_{cxy} \end{bmatrix} + \begin{bmatrix} \varepsilon_{xw} \\ \varepsilon_{yw} \\ \gamma_{xyw} \end{bmatrix} + \begin{bmatrix} \varepsilon_{x\Delta} \\ \varepsilon_{y\Delta} \\ \gamma_{xy\Delta} \end{bmatrix} \tag{14}$$

Fig. 3 shows the algorithm for determining the applied shear stress-strain response for the reinforced concrete membrane elements.

The strains caused by the crack opening and crack slipping are

$$\begin{bmatrix} \varepsilon_{xw} \\ \varepsilon_{yw} \\ \gamma_{xyw} \end{bmatrix} = \frac{w}{S_\chi} \begin{bmatrix} \cos^2\chi \\ \sin^2\chi \\ \sin 2\chi \end{bmatrix} \text{ and}$$

$$\begin{bmatrix} \varepsilon_{x\Delta} \\ \varepsilon_{y\Delta} \\ \gamma_{xy\Delta} \end{bmatrix} = \frac{\Delta}{2S_\chi} \begin{bmatrix} -\sin 2\chi \\ \sin 2\chi \\ 2\cos 2\chi \end{bmatrix} \text{ respectively.}$$

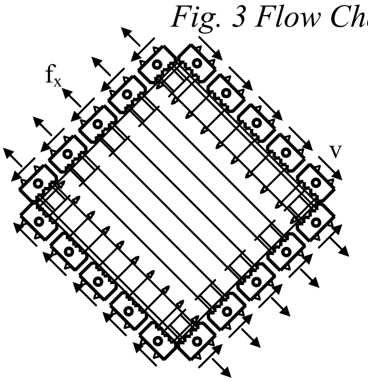


Fig. 4 Setup for PB series shear panels

The crack width and slip displacement, w and Δ taken as $S_\chi \varepsilon_{ci}$ and $S_\chi \gamma_{cij}$ respectively, if the crack spacing, S_χ is determined, where

$$\varepsilon_{ci} = \varepsilon_{c2}\sin^2\beta + \varepsilon_{c1}\cos^2\beta \,, \gamma_{cij} = (\varepsilon_{c1} - \varepsilon_{c2})\sin 2\beta$$

(normal strain and shear strain across cracks, by Mohr circle) and β is the difference between α and θ.

The total applied stress and shear capacity can be determined respectively.

$$\begin{bmatrix} f_x \\ f_y \\ v_{xy} \end{bmatrix} = \begin{bmatrix} f_{cx} \\ f_{cy} \\ v_{cxy} \end{bmatrix} + \begin{bmatrix} \rho_{sx} f_{sx} \\ \rho_{sy} f_{sy} \\ 0 \end{bmatrix} \tag{15}$$

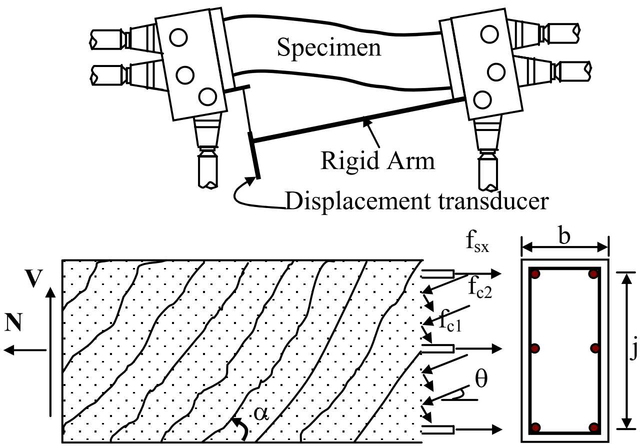


Fig. 5 Details of ST series beams

where ρ_{sx} and ρ_{sy} are the reinforcement ratio in x- and y- directions respectively, f_{sx} and f_{sy} are expressed from the equations (11) and (12) by putting ε_s equals ε_x and ε_y, f_x and f_y equal zero for the reinforced concrete membrane elements subjected to pure shear.

3. Prediction of Test Results

3.1. Shear strength and shear stress-strain relationship

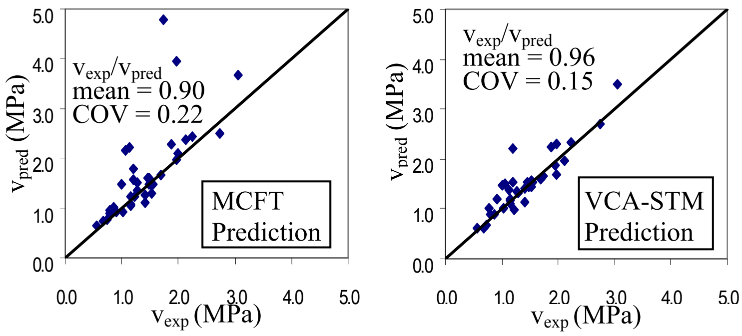


Fig. 6 Comparisons of experimental and analytical results for shear strength

The VCA-STM was verified by the test results of the 20 PB series panels (890 x 890 x 70 mm, details as shown in Fig. 4) and the 17 ST series beams (290 x 310 x 1625 mm, details as shown in Fig. 5) by Bhide & Collins (1989) and Adebar (1989) and respectively, [9, 10] with respect to the shear strength and shear stress-strain relationship.

These tests involved normal concrete shear panels subjected to combined axial load and shear (f_x:v ratios were ranging from –3 to 6). They contained different steel ratio (ρ_{sx} and ρ_{sy}) in x- and y- directions (ρ_{sx}: 1.78 - 3.49 and ρ_{sy}: 0 – 0.36 for ST series; ρ_{sx}: 1.09 – 2.02 and ρ_{sy} = 0 for PB series). From the predicted results, it can be observed that the VCA-STM predicted the shear strength of the test panels with a good accuracy (mean of test/predicted = 0.96 and COV = 0.15), which is shown in Fig. 6.

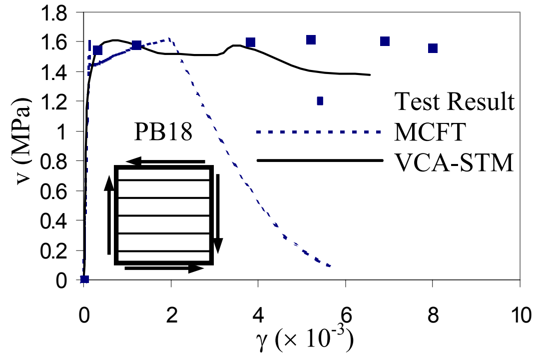


Fig. 7 Comparison of predictions of beam element

Fig. 7 shows the applied shear stress-strain relationship (v_{xy} Vs γ_{xy}) predicted by VCA-STM and MCFT comparing with the specimens PB18. The specimens PB18 was having the longitudinal reinforcement only and subjected to pure shear. It can be seen that the prediction given by the VCA-STM agrees very well with the experimental results, especially in the ascending part after appearance of cracking.

3.2. Angle of cracks and principal angle of crack concrete

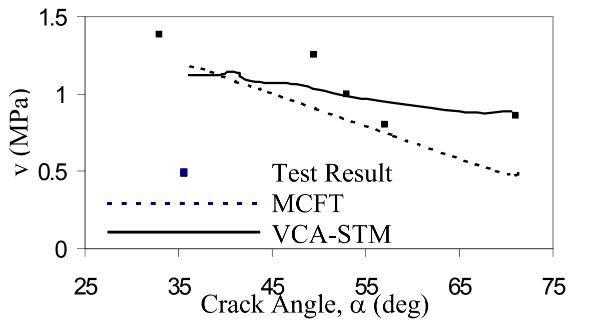


Fig. 8 Comparison of crack angle of panel PB21

Fig. 8 compares the observed and predicted crack angle of specimen PB21 (ρ_{sx} = 2.02 and ρ_{sy} = 0), which was loaded by combined shear and axial tension (f_x:v = 3.1). It can be seen that the observed pattern of crack angle is predicted quite well with VCA-STM along the whole progress. This agrees the observations that failure occurred at the appearance of secondary cracks for members with less or no stirrups.

4. Discussion

The inclination of initial cracking is normally determined from the condition of applied stress greater than 45 degrees in most beams considered. If the crack-angle is assumed to be 45 degree for FA-STM would give a trend of overestimation of concrete shear stress. Because the contribution of the normal stress component at cracks is smaller; and the solution converges up to relatively larger crack opening and slipping in larger fixed-angles. It is also noted that as the load is progressed, new cracks formed in a relatively lower inclination compared with an initial crack. On the other hand, MCFT assumed the crack-angle always equals the principal inclination of cracked concrete and started with a lower value of shear stress, however, the VCA-STM can separate crack angle and principal inclination of cracked concrete and give relatively more accurate results.

5. Conclusions

According to this research, the following conclusions can be drawn:

1. The variable crack angle softened truss model, summarized in this paper, is capable of accurately predicting the shear capacity of reinforced concrete members subjected to combined shear and axial load when comparing with the experimental results and it is simple to be used as simple as the MCFT.
2. The consideration of change of angle of cracks is proposed by the VCA-STM can describe the appearance of secondary of cracks of shear panels with less or no transverse reinforcement, which were subjected to combined shear and axial load.
3. As the shear panels subjected to pure shear were not totally isotropic materials when the steel ratio in x- and y- direction is not equal. The proposed shear stress of concrete (Eqs. (7) and (13)) satisfies the stress and strain conditions in Mohr circle and allows the change of crack angle for the entire load-deformation history.

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