

## Some aspects on safety of cantilevered and launched girders

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### Summary

Two popular methods of erecting bridges, cantilevering and launching, can lead, among other things, to a decrease in structural safety during construction. The first problem analysed is the possible overturning of a cantilevered girder on the pier and the second problem is cracking or breaking of a launched girder. A probabilistic approach was employed to both. The models comprised latent loads: the weight of the girder itself, loads affected by technical operations of the deck and wind action or the girder's weight, prestressing force, deviations of bearing levels and the strength of the steel/concrete used. Some of these were verified by original research undertaken on site. Second and third level reliability methods were used. The reliability index or probability of failure was used as the measure of safety. The results show the safety margins of such a structure during erection.

**Keywords:** concrete bridge, construction, cantilevering, launching, safety, reliability, random value, probability of failure

### 1. Introduction

#### 1.1. Definition of the problem

Two widely employed methods of erecting bridges are cantilevering and launching. Both are well tested and both have many advantages. But it is rarely mentioned that structural safety during erection can be visibly lower than after completion of a bridge. The most critical state arises, when the centre of a girder is supported on a pier before closing a cantilevered span. In this case, anxiety about overturning on the pier, due to a loss of stability is natural (Fig. 1). However, the question about safety during erection might be less obvious. When a girder is launched along the bridge's axis, errors in bearing levels lead to additional stress on a structure (Fig. 2). Usually the compression of concrete due to assumed deflection (settlements) is checked, but the problem of cracking is sometimes taken as less interesting. The following questions arise: what is the probability of a cantilevered girder overturning and what is the probability of a launched girder cracking or breaking – both under the assumed and real loads and production errors.

#### 1.2. Stability of a cantilevered girder

The method of symmetrical cantilevering involves the simultaneous concreting of girder segments, when its centre is supported on a pier and is also held up by a temporary support, placed close to the permanent one [1]. The temporary support is used to resist an unknown (in practice) overturning moment at the pier, caused by the loads created mainly due to a real asymmetry in the distribution of the weight [2, 3] of the girder and also by the technical loads, wind etc. (Fig. 3). The load capacity of the temporary support plays a fundamental role in the safety of girder being erected, because of the possibility of the structure collapsing and catastrophic results of such an accident. The limiting state [4, 5] can be described as loss of stability due to the overturning moment ( $M$ ) exceeding the 'moment-resistance' ( $M_r$ ) of the support. The values of these loads are usually set arbitrarily, but in fact, all of them are random quantities, which leads to a random load on the temporary support.



Fig. 1 Cantilevered girder – Bridge over the Vistula near Torun, Poland



Fig. 2 Launched girder – Viaduct in Wroclaw, Poland

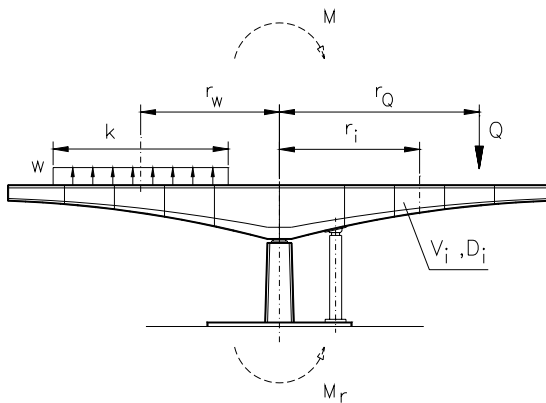


Fig. 3 Scheme of the definition of the limiting state – Cantilevered girder

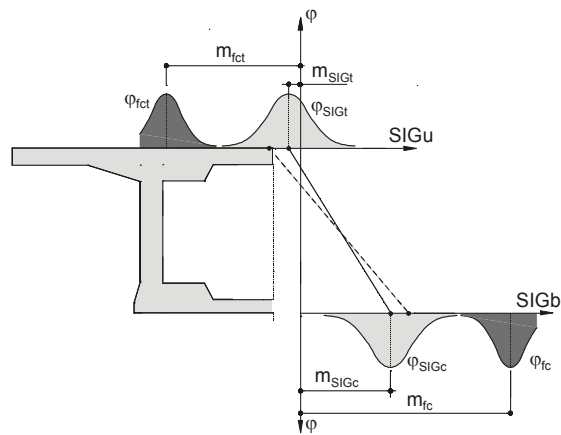


Fig. 4 Probability distributions of normal stress and strength of concrete – Launched girder

For this task the limiting state function  $Z(x)$  is defined in the following way (the argument  $x$  is a random vector  $\mathbf{X}$ ):

$$Z(x) = Z(M_r, V_1, \dots, V_n, D_1, \dots, D_n, r_1, \dots, r_n, r_Q, Q, w, k, r_w) = M_r - \left( \sum_{i=1}^n V_i D_i r_i + Q r_Q + k w r_w \right) \quad (1)$$

where (Fig. 3):

$n$  – number of segments of the cantilevered girder,

$V_i$  – volume of segment  $i$ ,

$D_i$  – specific gravity of segment  $i$ ,

$Q$  – force modelling technical loads on a deck,

$w$  – value of wind action,

$k$  – length of girder affected by wind action,

$r_i$  – distance from the segment's midpoint to pier,

$r_Q$  – distance from moving force  $Q$  to pier,

$r_w$  – distance from the resultant of wind action  $w$  to pier,

$M_r$  – limiting moment, characterising the load capacity of the temporary support under compression.

It is assumed, that the random variables are:  $V_i, D_i$  for  $i = 1, \dots, n$  and  $M_r, w, r_w, r_Q$ . But  $r_i$  for  $i = 1, \dots, n$ , are treated as deterministic quantities;  $Q$  and  $k$  are taken as parameters.

A safe state is obtained when  $Z(x) > 0$ . The task is to calculate the probability of failure i.e. exceeding the limiting state – collapse of girder.

### 1.3. Safety of a launched girder

Most sections of a launched girder are subjected to completely variable internal forces. If we want compression throughout launching, substantial prestressing of proper layout is required. The values of internal forces and the stresses and safety of the structure are mainly dependent on a static system (length of spans, presence of additional supports), as well as the amount and position of prestressing. But there are some factors, which are actually unknown. They are: real levels of bearings, values of the prestressing force, real volume and weight of the girder. Also, when the safety is defined as the resistance of a girder to cracking or breaking, the real strength of steel and/or concrete plays an important role.

Assuming that most factors are random, we can find the probabilities of:

- exceeding the acceptable stress level in concrete - tension (cracking of concrete),
- exceeding the acceptable stress level in concrete - compression
- exceeding the limiting state defined as breakage of a girder due to tension (breaking of bars and/or tendons).
- exceeding the limiting state defined as breakage of a girder due to compression (crushing of concrete),

The limiting state can be defined for instance as follows:

$$Z_t(x) = -f_{ct} + SIG_t \quad (2)$$

$$Z_c(x) = +f_c - SIG_c \quad (3)$$

Where (all are random variables):

$Z_t, Z_c$  – safety margin of tension and compression in concrete, respectively,

$f_{ct}, f_c$  – strength of concrete in tension and compression, respectively,

$SIG_t, SIG_c$  – extreme stresses in girder (tension and compression).

The safety margins are presented in Fig. 4, in which a temporary over-pier cross section of a launched girder is analysed. Tension appears in the top slab and compression in the bottom. Safety margins can be noticed. (Here  $m$  denotes mean value,  $\varphi$  - probability distribution,  $SIG$  - normal stress; indexes:  $_u$  – upper fibre,  $_b$  – bottom fibre). Again, the task is: what are the probabilities of exceeding the limiting states (or what are the reliability indexes).

## 2. Methods

The problems mentioned above were analysed using the concept of reliability. The tasks can be resolved by *second level* methods, where only the two first statistical moments are needed. Then, the measure of safety is a reliability index  $\beta$ . Another way to resolve the tasks is to use *third level* methods, where full information about random parameters, hidden in a probability distribution, is taken into account. Then, the safety measure is the probability of failure  $p_F$  or corresponding reliability index  $\beta_G$ . Second level methods [5] are called distribution free methods. One possible approach is the use of Cornell's reliability index, given by the formula (4).

$$\beta_C = \frac{E[Z]}{\sqrt{Var[Z]}} \quad (4)$$

Where  $E[Z]$  is the expected value and  $Var[Z]$  is the variance of variable  $Z$ .

When using third level methods, the probability of failure is employed as a reliability measure:

$$p_F = \int_{\{Z(\mathbf{x}) < 0\}} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \quad (5)$$

Here  $f_{\mathbf{X}}$  denotes the multidimensional joint probability density function of the random vector  $\mathbf{X}$ . In most practically interesting cases the exact value of  $p_F$  is extremely difficult to obtain. In such a case approximate methods should be applied. Among them the FORM (first-order reliability method) and the SORM (second-order reliability method) are most commonly in use [6]. Both of

them were employed. After transforming variables  $X$  into the standard normal space, the resultant probability of failure  $p_F$  and reliability index  $\beta_{G \text{ FORM/SORM}}$  can be found:

$$\beta_{G \text{ FORM/SORM}} = -\Phi_0^{-1}(p_{F \text{ FORM/SORM}}) \quad (6)$$

where  $\Phi_0$  denotes the one-dimensional standard normal cumulative distribution function.

Whenever it was possible, probabilistic data were directly employed in the solution presented, but sometimes it was reasonable to use Monte Carlo techniques.

### 3. Data

Knowledge concerning fluctuations of all variables is necessary for proper assessment of structural reliability. Some of the statistical data presented here were measured originally on site, during construction of bridges in Poland. The rest of the data were found in various other sources. The data were subjected to statistical analysis. Loads with their statistical parameters employed here are specified in Table 1 and Table 2.

Table 1 Statistical data concerning analysis of the stability of a cantilevered girder

Random variable	Distribution type	Expected value	Standard deviation	Source of data
$M_r$ – Limiting moment of capacity	Normal	20050 kNm	2005 kNm	Others / Design Documentation
$V_i$ – Volume of segment	Normal	Individual	Individual	Original research
$D_i$ – Unit weight of concrete	Normal	26 kN/m <sup>3</sup>	0.39 kN/m <sup>3</sup>	Original research
$r_i$ – Situation of midpoint of segment	Deterministic	Individual	-	
$r_Q$ – Distance to force $Q$	Rectangular	0.0	26.00 m	Assumed
$Q$ – Moving force	Deterministic	50 kN	-	Others
$w$ – Wind action defined by wind speed	Gumbel	26.0 m/s 16.3 m/s	4.1 m/s 2.1 m/s	Others
$k$ – length of girder affected by wind	Deterministic	30.00 m		Original research
$r_w$ – distance from the resultant of $w$ to pier	Rectangular	0.0	17.3 m	Assumed

All the data gathered in Table 1 were fitted to a cantilevered girder of length 90.00 m (each cantilever 45.00 m) built in Opole, Poland. Most of the original data gathered in Table 2 were collected during construction of a 430 m long launched girder, in Wroclaw, Poland.

The expected value of the limiting moment  $M_r$  was calculated using a deterministic approach together the following loads:

- asymmetrical (one-side) overload reflecting overweight of all segments on one cantilever, equal to 3 % of its weight
- concentrated load  $Q = 50$  kN located at the tip of one cantilever,
- wind load  $w = 1,8$  kN/m on one cantilever.

All these loads were multiplied by load factors according to Polish Code [7].

## 4. Results

### 4.1. Safety of a cantilevered girder

The reliability index and probability of failure obtained were:

Table 2 Statistical data concerning analysis of the safety of a launched girder

Random variable	Distribution type	Expected value	Std. deviation or coefficient of variation	Source of data
Cross section dimensions	Log-Normal	Individual	1.0 % 5.0 %	Original research
Specific gravity of concrete	Normal	27.0 kN/m <sup>3</sup>	1.5 %	Original research
Cross section area of one prestressing strand	Normal	151.00 mm <sup>2</sup>	0.722 mm <sup>2</sup>	Original research
Cross section area of one reinforcing bar	Normal	113.1 mm <sup>2</sup>	1.5 %	Others
Prestressing force	Normal	Individual	3.5 %	Others
Elasticity modulus	Normal	41.0 GPa	3.0 %	Others
Deflection of launching axis (settlements of piers)	Log-normal	9.0 mm	20 %	Original research
Deflection of launching axis (settlements of temporary supports)	Log-normal	17.0 mm	25 %	Original research
Strength of concrete under tension	Normal	- 4.216 MPa	0.816 MPa	Others
Strength of concrete under compression	Normal	78.30 MPa	7.348 MPa	Original research
Strength of prestressing steel	Normal	1926.94 MPa	32.084 MPa	Original research
Strength of reinforcing steel	Normal	565.90 MPa	20.932 MPa	Original research

$$\beta_C = 4.93$$

$$\beta_{G \text{ FORM}} = 4.88 \text{ with } p_F = 5.26 \times 10^{-7},$$

It is worth noticing that both values of  $\beta$  are similar and appropriately large. Furthermore, parametric analysis was performed, and some modifications of the assumptions were made mainly concerning the stochastic independence of random variables and value of force  $Q$ .

## 4.2 Safety of a launched girder

The results are presented in Table 3.

Table 3 Minimal values of  $\beta_{G \text{ FORM}}$  indexes

Analysed limit state	Launching with temporary supports	Launching without temporary supports
Exceeding the strength of concrete under tension (cracking of concrete)	0.827	3.861
Exceeding the strength of concrete under compression	8.642	8.143
Exceeding the limiting state defined as breakage of the girder due to tension (breaking of bars/tendons)	3.808	27.515
Exceeding the limiting state defined as breakage of the girder due to compression (crushing of concrete)	9.802	9.373

As can be noticed, the most unfavourable results concern cracking of concrete under tension. Reliability indexes for the other factors are higher, especially for the limiting state defined as breakage of the girder due to tension, when launched without temporary supports.



## 5. Conclusions

### 5.1. Comments regarding the stability of a cantilevered girder

- The reliability indexes  $\beta$ , under the assumptions made in the task, are close to 4.9, almost independently on the method employed. They correspond to small probabilities of failure. This indicates that the loads and factors used in practice based on safety codes assure a sufficiently large safety margin.
- The reliability index immediately decreases, when the force  $Q$  increases (not shown here). Sometimes, the temporary support is checked considering a machine moving on the deck. It is important not to use heavier machines in reality, because of a significant decrease in the reliability index.
- The sensitivity factor  $\alpha$  referring to the volume of a segment was found to be the highest (not shown here). It means that loads on the temporary support, related to variability of thickness are substantial. So, permanent monitoring of the variability of thickness is recommended to avoid overloading the temporary support.
- The results can be treated as representative, because of the similar technology of concreting employed, and the quality of examined objects compared to other modern bridges.

### 5.2 Comments regarding the safety of a launched girder

- Unexpected settlement of temporary supports (errors in the bearing level) leads to a substantial decrease in the reliability index in the aspect of the cracking of concrete.
- When launching without temporary supports (at the same tension stress restrictions) the reliability indexes are higher.
- It is difficult to ensure satisfactory stress level by checking the cracking of concrete (at limited compression). The requirement of full compression during launching leads to a substantial increase in prestressing quantities and – of course – its costs.

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