Nonlinear Analysis of Discontinuity Regions by the Strut-and-Tie Method

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Summary

The Strut-and-Tie Method (STM) is a code-worthy methodology for the design of D-regions in structural concrete. However, current strut-and-tie design methodologies do not address serviceability requirements necessary for certain D-Regions, such as deep beams and squat walls.

This paper presents a design procedure for explicitly satisfying both strength and serviceability requirements. The suggested computational approach accounts for the nonlinear stress-strain characteristics of both struts and ties. The feasibility of the approach is discussed using the experimental results of a deep beam.

Keywords: strut-and-tie method; strut-and-tie models; truss models; discontinuity regions; disturbed regions; structural concrete; deep beams; nonlinear analysis; design method.

1. Introduction

The Strut-and-Tie Method (STM) provides a consistent and code-worthy methodology for the design of structural concrete regions in which there is a nonlinear variation in straining, commonly referred to as D-(Discontinuity) regions. The STM involves the idealization of an internal truss in the cracked body of concrete, i.e., the strut-and-tie model, to carry the loading through the D-region to its supports or boundaries. The selected internal truss is dimensioned by providing the tension members sufficient reinforcement to carry the tensile truss forces and ensuring that the compression members and nodal zones are sufficiently strong to support the corresponding truss forces. The rules for calculating the strength of these components are defined in codes of practice.

Current strut-and-tie design methodologies do not address serviceability requirements. Proposed by Schlaich et al. [1], the current widely used approach to consider serviceability requirements is implicit, based on the premise that both serviceability and ultimate limit states of a D-region subjected to loading will be satisfied if the orientation of each strut-and-tie model component is selected to align with the principal tensile or compressive direction according to the elastic distribution of stresses. Schlaich proposed that the directions of the struts and ties be within 15 degrees of the elastic solution [1]. An advantage in this approach is that only one model is required to satisfy both strength and serviceability limit states, but this approach does not provide designers with the often needed flexibility to use more traditional reinforcement orientations (i.e. horizontal and vertical), nor does it address how to design statically indeterminate structures.

This paper reflects an ongoing research work to address the serviceability issues in the STM. A procedure for design satisfying both strength and serviceability requirements was identified and is presented herein. This procedure is explicit, based on the use of computational models accounting for nonlinear stress-strain characteristics of struts and ties to estimate the response quantities at service load level. The relevant service-load quantities are then directly checked to meet the serviceability criteria defined in the codes of practice. The feasibility of this approach is discussed using the results of an experimentally tested deep beam.

2. Load-Deformation Analysis Based on the STM

Load-deformation analysis based on the STM involves nonlinear static analysis of strut-and-tie models representing load-carrying system of the structure under proportional loading. The analysis satisfies equilibrium, stress-strain relationships, and strain compatibility within the strut-and-tie model components at each loading stage. The response obtained from this analysis accounts for nonlinear stress-strain characteristics of the struts and ties. Advantages of this analysis type include utilizing simpler models than those of finite element analysis and focusing the response results on the main load-carrying members.

There have been several research works utilizing load-deformation analysis of strut-and-tie models. Research purposes range from simply predicting better strengths than those predicted by conventional STM to evaluating the redundant forces in statically indeterminate strut-and-tie models. Hwang and Lee [2], for example, proposed a strut-and-tie model for interior beam-column joints and used load-deformation analyses of the model to predict the shear strengths. In another example, Yun [3] demonstrated the use of nonlinear analysis of strut-and-tie models to predict the entire load-deformation response under monotonic loading. In yet another example, Sundermann and Mutscher [4] used load-deformation analysis of strut-and-tie models to estimate the support reaction forces of an externally statically indeterminate deep beam. In the last example, the strut-and-tie model geometry is adjusted at each loading stage such that the total complementary energy of the system is a minimum. An attempt to simulate the hysteresis response of reinforced concrete structures using strut-and-tie models was also made [5].

In general, appropriate strut-and-tie models, effective widths of struts and ties, constitutive models of struts and ties, and effective strengths of struts and ties are the critical parameters affecting the results of load-deformation analysis. Basic assumptions usually employed are listed below; these assumptions may be added or removed, depending on the complexity of computational models selected.

1. The use of strut-and-tie model is valid for the entire loading stages.
2. The primary modes of failure are yielding of ties, crushing of struts or nodal zones, and diagonal splitting of struts. Crushing of nodal zones is identified as failure at the strut ends.
3. The stress-strain relationships of struts and ties can be characterized and are known.
4. The effective widths of struts and ties can be characterized and are known.
5. The stress distribution across the effective widths is uniformly distributed.
6. The force in struts and ties at any loading step can be estimated as the product of stress and effective width.
7. The structure is in free-stress state at the beginning of the analysis.
8. Deformation in nodal zones is neglected.
9. Small deformation theory is employed.

3. Load-Deformation Response of a Deep Beam

The use of nonlinear analysis using strut-and-tie models for predicting the entire load-deformation response is illustrated in this section using a deep beam example. The deep beam under consideration is the SD-1 test beam of an experimental program conducted by Lee [6]. It is simply supported at the ends and is singly loaded on the top at the midspan. The center-to-center span is 2800 mm, the height is 1000 mm, and the width is 200 mm. Longitudinal bars consist of 4 layers of 3 #6 (19 mm) bars. Two-legged stirrups of 10M (10 mm) spaced at 200 mm serve as the transverse reinforcement. Longitudinal bars of 10M spaced at 300 mm at each face of the beam provide for horizontal distributed reinforcement. The dimensions, reinforcement details, and loading configuration are shown in more detail in Fig. 1.

The concrete compressive strength, $f_c^{'}$, is 33.5 MPa. The steel yield strength, f_y, is 498 MPa and 529 MPa for the #6 bars and 10M bars, respectively.

The failure load, P_u, was reported to be 1935 kN. The corresponding midspan deflection was 12 mm. The failure was identified by crushing of one of support regions followed by anchorage loss of the longitudinal reinforcement.

Three strut-and-tie models shown in Fig. 2 are selected to represent possible load paths of the point load to the supports. The first model (Fig. 2(a)), designated as M1 Model, consists of struts *ABC* and *DE* connected by tie *BD* representing the shear reinforcement and horizontally equilibrated by tie *ADE* representing the longitudinal reinforcement. In the second model (Fig. 2(b)), a direct strut *AC* from the point of loading to the support is added to the load-carrying system of M1 Model. The second model is a statically indeterminate strut-and-tie model and is designated as M2 Model. The third model, not illustrated here, consists of only a direct strut *AC* from the point of loading to the support that is equilibrated by tie *ADE*. Designated as M3 Model, the third model represents a load path that ignores the presence of transverse shear reinforcement.

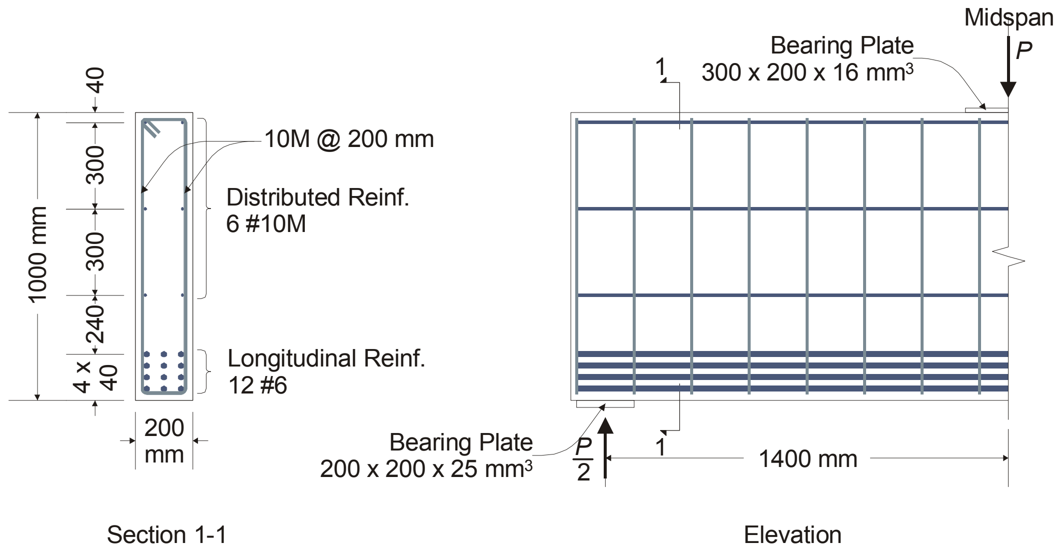


Fig. 1 Description of SD-1 test beam (adopted from [6])

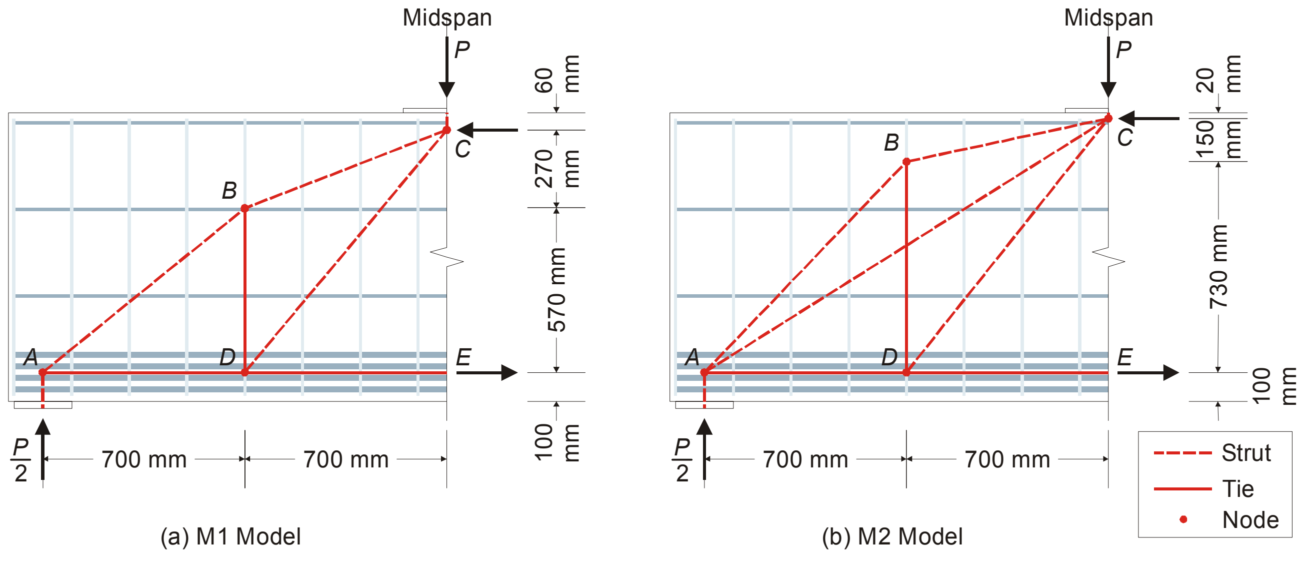
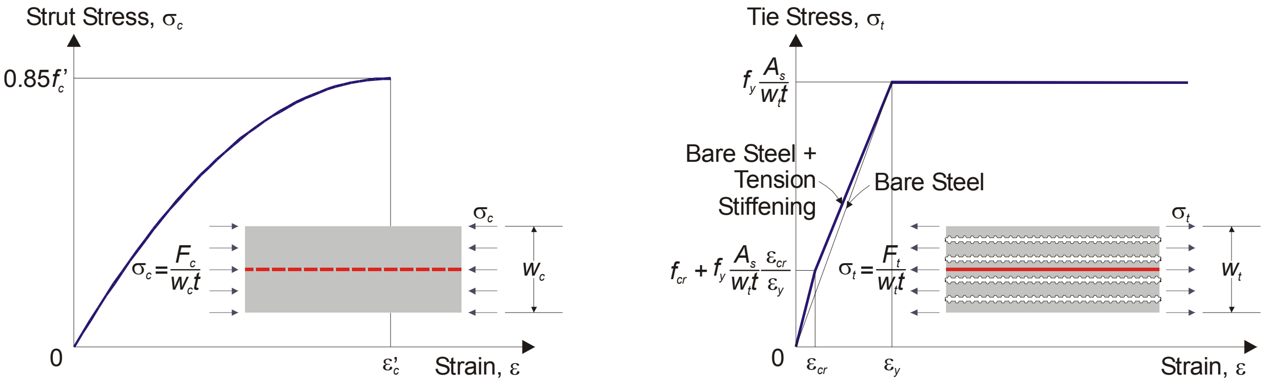


Fig. 2 The first two strut-and-tie models selected

Simple stress-strain relationships of struts and ties are employed for all models and are shown in Fig. 3. The stress-strain curve for the struts follows the average curve obtained from concrete test cylinders with the ordinates being scaled down such that the peak strength equals $0.85f_c^{'}$ to represent the strut effective strength (Fig. 3(a)). The reinforcing steel is assumed to govern the stress-strain relationships of the ties; simple tension stiffening relationships are considered and are assumed to be only significant prior to the yielding of the steel (Fig. 3(b)). The notations displayed in the figure, but not mentioned anywhere else in the discussion are as follows: A_s = tie reinforcement area; F_c = strut force; F_t = tie force; t = deep beam width; f_{cr} = concrete cracking stress; $\varepsilon_c^{'}$ = concrete strain when strut stress reaches peak stress; ε_{cr} = concrete cracking strain; ε_y = steel reinforcement yield strain.

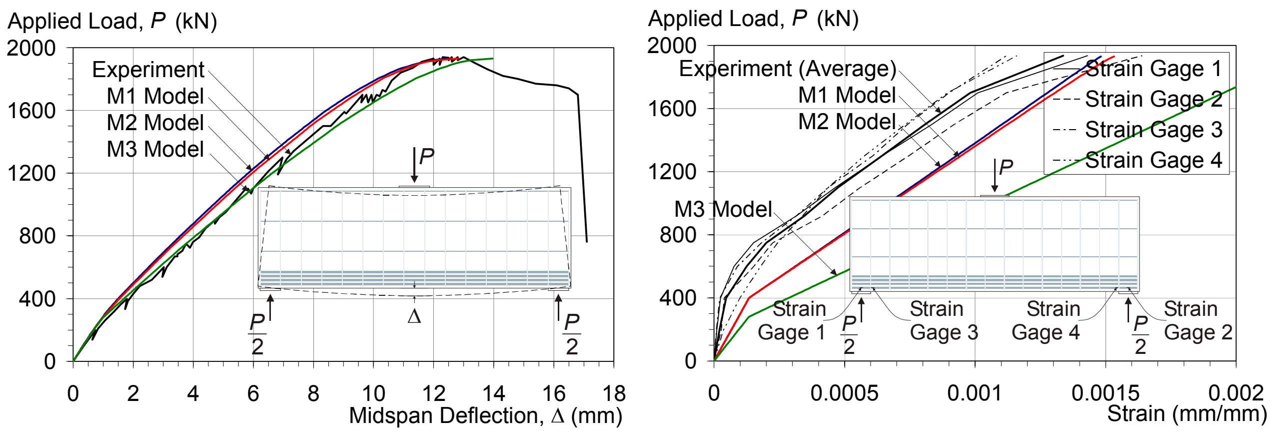


(a) Struts (b) Ties

Fig. 3 Utilized stress-strain relationships

The effective widths of the struts, w_c, are selected such that the stress at the collapse load (P_u = 1935 kN) is equal to $0.85 f_c^{'}$. For M1 Model, the corresponding force distribution in the struts at P_u = 1935 kN is determined by statics. For M2 Model, the corresponding force distribution is determined in proportion to the relative elastic stiffness of the load paths. The effective widths of the longitudinal and transverse ties, w_t, are assumed to be 200 mm and 700 mm, respectively.

The nonlinear analyses for the three models are performed using CAST (Computer Aided Strut-and-Tie) program [7]. During analysis, the geometry of these strut-and-tie models is fixed. The analyses are terminated when the applied load, P, reaches P_u = 1935 kN.



(a) Midspan Deflection (b) Longitudinal Steel Strain Near Support Regions

Fig. 4 Analytical and measured response

Fig. 4(a) plots the experimental and analytical midspan deflections as a function of the applied load, P. Fig. 4(b) compares the measured and predicted longitudinal strains of the ties near the support regions. It is shown in Fig. 4(a) that multiple strut-and-tie models with consistent and relatively simple properties can match the experimental result at global level. However, as shown in Fig. 4(b), only strut-and-tie models that realistically represent load-carrying systems of the deep beam give a good agreement with the experiment data at member level. For M3 Model, the straining of longitudinal tie is uniform along the span of the beam. Therefore, the midspan deflection and longitudinal tie strains in particular are much overestimated. For M1 and M2 models, the predicted longitudinal strains still overestimate the measured strains because some lateral confinement that exists in the longitudinal ties near the support regions due to reaction forces is not accounted for in the analysis. In addition to recognizing the appropriate strut-and-tie models, it is also observed here that satisfactory agreement between experimental data and analysis results can be reached if effective strengths observed from experiment and those selected for the analysis are within the same order. This observation becomes the basis of the design procedure described next.

4. Design for Strength and Serviceability Based on STM

The preceding example suggests the feasibility of using load-deformation analysis of strut-and-tie models to estimate response quantities at service load level. In combination with the STM design procedure for strength, this leads to a design procedure that explicitly satisfy both strength and serviceability requirements since the relevant service-load quantities can be directly checked to meet the serviceability criteria defined in the codes of practice.

The description of the design procedure is as follows:

Step 1. Design for strength. This design stage includes the following:

- *a.* Defining the D-region under consideration and evaluating the boundary and body forces,
- *b.* Sketching a strut-and-tie model and solving for the truss member forces,
- *c.* Selecting the ordinary reinforcing steel and prestressing steel that are necessary to provide the required tie capacity and ensure that they are properly anchored in the nodal zones,
- *d.* Evaluating the dimensions of the struts and nodes, i.e., selecting the effective widths of the struts and constructing the shapes of the nodal regions, such that the capacity of these components is sufficient to carry the design force values, and
- *e.* Ensuring sufficient ductility capacity in the D-region, e.g., by providing distributed reinforcement.

Step 2. Check for serviceability. This design stage includes:

- *a.* Selecting appropriate stress-strain relationships for dimensioned struts and ties and scaling down the curve points such that the peak stress is equal to the corresponding effective strength used in Step 1,
- *b.* Performing nonlinear analysis,
- *c.* Determining response quantities at service load level, and
- *d.* Checking the relevant response quantities against the serviceability criteria.

5. Discussion

As shown in Section 4, the proposed design procedure is in line with the present design procedure for strength. It only needs several steps beyond those required in design process for strength. It can be used during preliminary determination or during final evaluation of strength and stiffness.

It is recognized that the nature of nonlinear analysis involved in the design procedure makes it essential to use a computer-based STM to perform a design. However, it is well recognized that the application of STM for strength design is usually complicated by the need to perform iterative and time-consuming calculations and involves extensive graphical representations of strut-and-tie models [8]. As a result, computer-based STM tools equipped with graphics, analysis, and design tools, such as CAST [7], become essential to enable the designer to focus on the selection and design of the idealized load-resisting trusses. Furthermore, performing load-deformation analysis is becoming widely accepted in practice for evaluation of newly designed structures, to ensure that the structures are able to reach the design load according to the intended mode of failure.

As mentioned earlier in Sections 2 and 3, satisfactory design using the design procedure depends on selection of appropriate strut-and-tie models, in addition to the selection of effective widths, stress-strain relationships, and effective strengths of struts and ties. For complex D-regions, elastic stress distribution can still be a good start for identifying the load-carrying system, i.e., the strut-and-tie models. The explicit evaluation at serviceability limit state removes the deviation limitation of arrangement of strut-and-tie models according to elastic solution and allows the use of strut-and-tie models that are closer to the ultimate condition to aim at a more efficient design.

Critical areas for improvement of the design procedure presented in this paper are (1) characterization of appropriate stress-strain relationships and effective strengths of struts and ties and (2) estimation of relative stiffness of struts and ties in externally and internally statically indeterminate strut-and-tie models to obtain reasonable force distribution required in design step for strength. Current limited experience suggests that the results of using relative stiffness based on elasticity correspond well with those from load-deformation analysis accounting for nonlinear stiffness characteristics of struts and ties.

6. Conclusion

Nonlinear analysis of discontinuity regions using strut-and-tie models is an extended version of the Strut-and-Tie Method (STM) for strength design. This analysis accounts for nonlinear characteristics of strut-and-tie model components. This analysis has been utilized in research studies for a number of reasons, such as for predicting load-deformation response and evaluating the redundant forces in statically indeterminate strut-and-tie models. In this work, nonlinear analysis of strut-and-tie models is used to predict load-deformation response at service load level.

A design procedure for design explicitly satisfying strength and serviceability requirements is presented. This procedure makes use of parameters used for strength design in combination with load-deformation analysis to estimate service-load response quantities for check required to meet the serviceability criteria. The feasibility of the approach is shown by an example of deep beam problem. The design process is in line with that for strength and requires only several steps beyond calculation steps for strength. This design procedure can be used in preliminary or final design.

7. Acknowledgments

This work is being supported by a CAREER Award of the National Science Foundation (CMS 0092668) to the second author. This support is gratefully acknowledged. The views expressed do not necessarily represent those of the organization.

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