Precast floor diaphragm action to ensure the structural integrity of multistorey buildings

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Summary
The objective of the paper is the tying of precast floors necessary to insure the integrity of the entire framed structure. Three structural functions are dependent upon the tying reinforcement, these are:

- diaphragm action of the floors
- composite action of floor slabs and beams
- alternative action model (in an accidental situation)


Keywords: precast floor slabs, floor diaphragms, floor membranes, precast composite structures, mechanism of interaction in a composite structure

1. Introduction

In the present European practice in construction of framed multistorey precast buildings there is a tendency to reduce height of the cross section of beams, which have the inverted T shape and together with the floor slabs can be designed as the composite structure. Small spaces left for the in situ concrete and large spans of the individual precast units justify the detailed design of tying reinforcement (Fig.1) with account for global and local functions which are (in first of all):

- diaphragm action (under in-plane loadings),
- composite action (under out-of-plane loadings),
- alternative model action (aimed at prevention against progressive collapse – in case of an accidental situation).

The point of author approach is that it is not sufficient to design the tying reinforcement according to general requirements like the ones settled by Eurocode 2 but it is necessary to determine the dimensions on base of calculation results carried out for each of named above model of action. The principles of these calculations are shortly presented here below.

Two types of floor units are mostly applied to create the composite beam – floor structures, these are hollow core slabs or double – tee floors. The heights of the cross sections of the named units may amount even 500 mm and 1000 mm respectively and the length up to even more than 30 m.
Fig. 1 Types of reinforcement provided to ensure the diaphragm and composite action of the precast floor

1 – continuous steel bars along cast in situ the tie beams
2 – ties across the vertical interfaces
3 – ties across the horizontal interface
4 – reinforcement (a mesh) located in structural screeding

2. Diaphragm action

Tying reinforcement placed in the perimeter tie beams and in the internal ones should be dimensioned not only with account for the strength (as it is commonly recommended in the majority of international and national documents) but with account for additional displacement limitations. The background principles and the complete calculation methodology (together with calculation examples) derived by author and K.S. Elliott have been published in [1] [3]. For the static conditions the formulae (1) and (2) were proposed, based on the relationships according to shear – wedging approach. The tensile force $F_{tv}$ imposed due to shear force $V$ is then:

$$F_{tv} = \frac{V - V_{sk}}{n, \mu} = \frac{V_{j}}{n, \mu}$$  \hspace{1cm} (1)

where: $V_{sk}$ – part of the shear force transferred directly by the concrete in the connection (acting as a shear key)
$V_{j}$ – part of the shear force $V$ transferred by the longitudinal joint
$n$ – total number of the perimeter and internal tie beams, which can be reckoned as transferring the shear force $V$
$\mu$ – friction factor (observe, not friction coefficient)

According to [1] the tying reinforcement should be dimensioned to satisfy following shear displacement limitation:

$$\delta_n = \frac{F_{tv}}{K_t} + \delta_{ti} \leq \delta_{t\text{max}} = 0,5 \text{ mm}$$  \hspace{1cm} (2)

where: $K_t$ – axial stiffness of the tie bean (see [1])
$\delta_{ti}$ – initial crack width (see [1] and [3])
$\delta_{t\text{max}}$ – limiting crack width for aggregate interlock [1]
The condition (2) leads to application of relatively high values of \( \mu \), even higher than \( \mu = 5 \) (see calculation example in [1]). The limitation of the crack width influences the global shear stiffness of the floor diaphragm because the shear displacements of the individual longitudinal joints are negligible small. The very effective shear-friction mechanism was also observed in tests reported by Moustafa [5]. On their background in PCI manual [6] so called effective coefficient of friction \( \mu_e \leq 2,2 \) was recommended.

Subdivision of the total \( V \) force into parts \( V_{sk} \) and \( V_j \) can be done proportionally to the shear stiffnesses of both shear transferring components, see [1] and [3].

### 3. Composite action

In the newest draft of ENV document [7] following motto is expressed “In addition to satisfying the requirements of cross-section design, the analysis of precast concrete structures shall take into account the behaviour of the connections between elements”. This motto fits well just to the point below discussed, whereas the role of the stiffness \( C_c \) of the shear subjected connection has been here particularly emphasized. Cholewicki [2] presented a solution according to two-beams model, which describes the effects of the shear deformability of the interface connection on the distribution of the unit shear forces \( V'c \) and the sum of those forces \( V_{c\,max} \) along the half length of the beam (Fig. 3).

Shear force per unit length for the case of a simply supported beam, is:

\[
V'_{c\,max} = V_o \frac{\psi}{\alpha^2} \eta_{max} \tag{3}
\]

Sum of the unit shear forces along the half of the span of beam, (Fig. 3a), is:

\[
V_{c\,max} = 0,5 V_o \frac{\psi}{\alpha^2} L \eta_m \tag{4}
\]

where: \( \eta_{max} \) and \( \eta_m \) – coefficients (table 1)
Table 1 Coefficients $\eta_{\text{max}}$ and $\eta_m$

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Fig. 3 Shear stresses distribution upon the $\alpha L/2$ value

a) two-beam model of the composite structure,
b) diagram of unit shear forces $V_c'$.

$c_1$ – connection with stiffness $C_c$

Ratio $\frac{\psi}{\alpha^2}$ is:

$$\psi = \frac{a}{E_{c1}I_1 + E_{c2}I_2} \left( \frac{1}{E_{c1}A_1} + \frac{1}{E_{c2}A_2} + \frac{a^2}{E_{c1}I_1 + E_{c2}I_2} \right)$$  

(5)

Characteristic value $\frac{\alpha L}{2}$ which governs the magnitudes of coefficients $\eta'$ and $\eta_m$ is:

$$\frac{\alpha L}{2} = \frac{L}{2} \sqrt{\left( \frac{1}{E_{c1}A_1} + \frac{1}{E_{c2}A_2} + \frac{a^2}{E_{c1}I_1 + E_{c2}I_2} \right)} C_c$$  

(6)

where: $E_{c1}, E_{c2}$ – respectively, concrete elasticity modulae in parts ① ②,
$A_1, A_2$ – respectively, cross section areas of parts ① ②,
$I_1, I_2$ – respectively, moments of inertia of parts ① ②,
$a$ – distance between the central points of parts ① and ②.
$L$ – span of the beam

The basic interaction equation for the composite structure is then:

$$M_o = M_b + M_{fr} + V_{c_{\text{max}}} \cdot a$$  

(7)

where: $M_o$ – bending moment
$M_b$ – moment transferred by the beam
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\( M_{fl} \) – moment transferred by the part of the structure lying either above the horizontal interface or lying outside the beam if the vertical interface connections are assumed to be deformable.

The third part in equation (7) is the coupling moment or moment due to interaction. Examples of calculations show that if \( \frac{\alpha L}{2} \geq \frac{\alpha L}{2} \) the shear stiffness \( C_c \) has not meaningful importance for the \( V_{c, max} \) force and \( V'_{c} \) distribution of unit shear forces (Fig. 3b). Shear transfer mechanism is becoming more complex in case when the neutral axis of the composite section lies within the height of the floor units (Fig. 4).

![Fig. 4 Hollow core floors supported on beam of the inverted T section](image)

- 1 – position of the neutral axis of the composite structure
- 2 – ties led through the beam
- 3 – shear keys in the vertical edges of the beam
- 4 – holes in the beam

The recommendations concerning such design situations should be, in author opinion, revised because \( M_{fl} \) (equation (7)) can contribute in more meaningful way than according to very limited \( b_{eff} \) given in [3]. This case needs however further research.

4. **Alternative model action**

In case of an accidental situation which leads to complete failure of one structural member (column) the appearance of the alternative bearing system is expected. The framed system acting as a whole must be able to transfer, through the bridging over the damaged column, the extra loads on the surrounded columns.

So called global model for the numerical analysis should be applied in order to distribute the vertical force from that one column which is supposed to be totally damaged. The distribution of that force on the adjacent undamaged columns follows in two planes i.e. in the plane of main internal beam and in the one being perpendicular to the main beam.

Author has carried out a theoretical study which showed that

1. through modification of design principles of flexural reinforcement in the main beam and
2. through modification of design of the reinforcement placed in the cast in situ tie beam and the ties in floor (sign 2 on Fig. 1)

the considerable increase of the bearing capacity of the alternative bearing model can be achieved.
5. Conclusions

1. In case of modern precast framed systems there is a necessity of a very precise dimensioning and detailing of ties, because the floor elements have relatively large spans and the gapes left for the in situ grouting concrete are small.

2. The same ties fulfil very differentiated structural functions, author has discussed them on examples of three models and particularly emphasized the effects of the stiffness of connections (diaphragm and composite action models).

3. The recommendations for tying formulated in national documents and authorized also by Eurocode 2 should be treated as the general guidance and they need to develop towards a system of specific design requirements.

4. Properly designed ties projected through the horizontal and vertical interfaces may guarantee the effective interaction mechanism in a composite beam, quantitative evaluation of that mechanism can be done by means of equation (7) and with application of two-beams model.

5. The effective interaction mechanism can bring the technical and economical advantages (as e.g. avoidance of unnecessary structural screeding and higher bearing capacity of the composite beam).

6. By adequately designed ties it is possible to ensure “hidden features” of the precast floor diaphragm to sustain an accidental situation.

6. References