Deformations and stresses in concrete slabs on grade

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Summary

Although the structural behaviour of concrete slabs on grade has been studied for many years, most of the methods that are used nowadays to determine the slab thickness were elaborated during the first half of the previous century. As long as the slab remains in the uncracked linear elastic stage, the deformations and stresses, predicted by those models, are quite accurate. In practice nevertheless, damage to concrete slabs can often be observed. This damage might be partly due to the non-linear behaviour of both the concrete slab and the base material, which are not incorporated in the former calculation methods. However also other phenomena like drying and shrinkage of the concrete and friction between concrete slab and base material induce additional stresses in the slab. The latter phenomena have been studied in an experimental test program carried out at the Magnel Laboratory for Concrete Research of Ghent University. An attempt is made to simulate these effects by means of a finite element model.

Keywords: concrete slab on grade, moisture diffusion, shrinkage, curling, friction, modelling.

1. Experimental research

To get a better insight in the behaviour of concrete slabs on grade and in the parameters influencing this behaviour, an experimental test program was carried out. In a first phase, slabs submitted to the outside weather conditions were considered. In a second phase, the shrinkage behaviour of small-scale test specimens was observed. A third test series dealt with slabs in a wind tunnel. Finally, friction tests were carried out.

1.1 Slabs submitted to outside weather conditions

The possible influence of parameters like the concrete mix design, the reinforcement type, the surface finishing and the curing of the fresh concrete on the behaviour of concrete slabs on grade submitted to the outside weather conditions was studied experimentally. A detailed overview of the tests that were carried out and of the test results that were obtained during the first year was given in [1]. Deformation and vertical displacement measurements were finally carried out over a period of two years.

The deformation behaviour of the slabs clearly showed a seasonal dependency. During sustained warm and dry weather, the slabs showed curling. For periods with other weather types, a curvature in the opposite direction was noticed.

The type of surface finishing seems to influence the curvature of the slabs. Slabs for which the top surface was only treated with a brush show a pronounced convex curvature while slabs provided with a special top layer show higher curling.

1.2 Small scale tests

The shrinkage behaviour of several concrete types is studied by performing deformation measurements on prismatic test-specimens with dimensions 150mm x 150mm x 600mm. The prisms are stored horizontally in an air-conditioned room at 20°C and 60% relative humidity. Some of the prisms can exchange moisture with the environment through all their side faces while others have only their top surface unsealed. The latter specimens give a good approximation of the drying conditions that exist in a concrete slab on grade that can only loose moisture through its top surface.

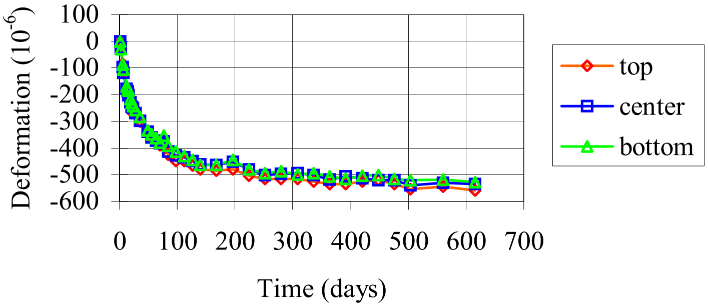

Fig. 1 Deformations measured on an unsealed prism

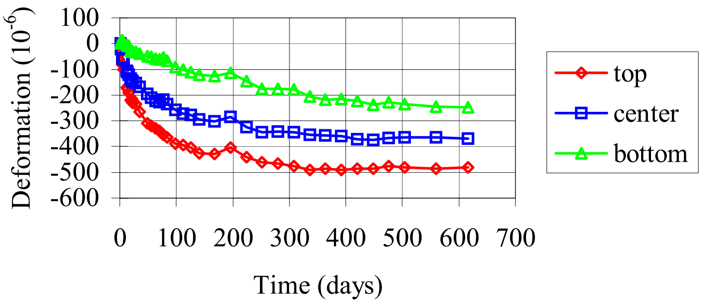

Fig. 2 Deformations measured on a partly sealed prism

Deformations are measured on the top surface of the specimens and at the centre and bottom level of their side faces. Figure 1 shows the results that were obtained on an unsealed prism. Figure 2 shows the results that were obtained on a partly sealed prism made with the same concrete.

All fibres on the outer surface of the unsealed prism show the same deformation. The prism only undergoes a uniform shortening. The deformations of fibres on the outer surface of the partly sealed prism are dependent on the distance between the fibre and the top surface. While the top fibres show a shrinkage deformation similar to that of the unsealed prism, the shrinkage of the other fibres decreases towards the bottom of the prism. Besides a uniform shortening, this prism also shows a curvature. The outer ends of the prism move upward relative to its centre part.

1.3 Wind tunnel tests

To study the influence of air-movement on the drying of a concrete slab, wind tunnel tests were carried out. Hereto slabs with a length of 10m, a width of 1m and a thickness of 0,15m were submitted to airflow over their top surfaces. The wind speed ranged between 3m/s and 10m/s but was kept constant for each slab tested. Where the prisms in the small-scale tests can move freely, the slabs in the wind tunnel are in contact with the formwork in which they were cast. In this way, the influence of friction between concrete slab and base is taken into account. The airflow is applied immediately after a slab is cast. The first hours after casting, the appearance of plastic shrinkage cracks is monitored.

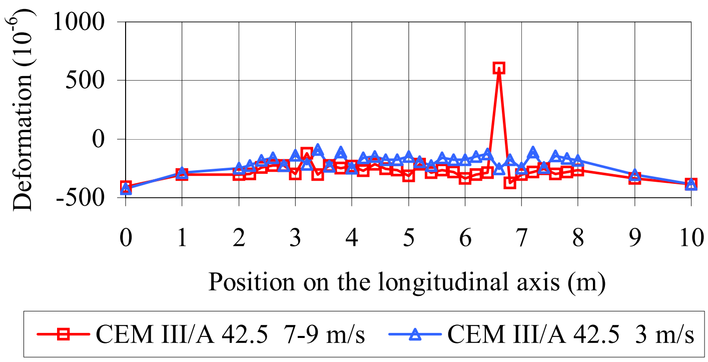

Fig. 3 Deformations measured along the longitudinal axis of the slab surface

One day after casting, measurement points are placed on the top surface of the slab. These measurement points allow the registration of the concrete deformation at the slab surface and of the vertical displacement of the slab as a function of time. As an example, figure 3 shows the concrete deformations measured 28 days after casting along the longitudinal axis at the top surface of two similar slabs tested at different wind speeds.

The air speed seems to influence the risk of plastic shrinkage cracking.

Slabs tested at wind speeds above 7m/s are vulnerable to plastic shrinkage cracking while slabs tested at wind speeds below 3m/s showed almost no cracking. In figure 3, the position of a plastic shrinkage crack, present in the slab tested at a wind speed between 7m/s to 9m/s, is clearly visible by the peak value in the measured deformations.

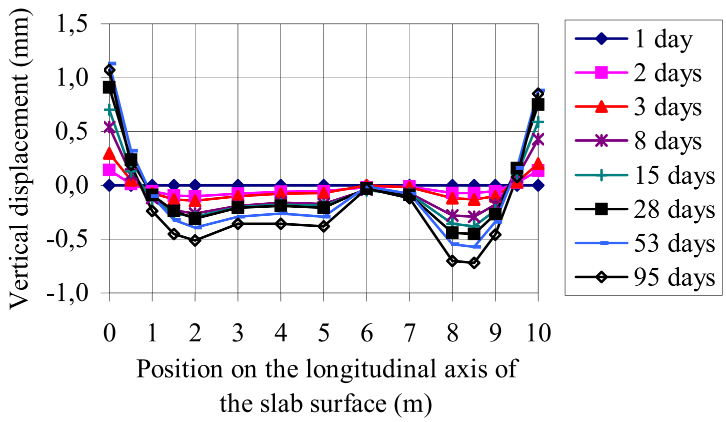

Fig. 4 Vertical displacements along the longitudinal axis of the slab surface

Furthermore it can be seen that the deformation of the slabs is less restrained near their outer ends where the measured deformations are higher than in the centre part of the slabs.

The vertical displacements along the longitudinal axis of the slab tested at the highest wind speed are given in figure 4 as a function of time.

The curling effect near the outer ends of the slab is clearly visible. Also in the region of the plastic shrinkage crack the curling effect becomes visible as the crack widens.

In an attempt to avoid plastic shrinkage cracking at high wind speeds the effect of external and internal curing and of polypropylene fibres (0,9 kg/m³, length 20mm and diameter 16µm) were tested. Where the use of curing had a favourable effect on the cracking behaviour of the slabs, the polypropylene fibres seemed to have no effect at all in the tests considered. More details about the small-scale tests and wind tunnel tests can be found in [2].

2. Finite element model

The experiments mentioned above and the damage encountered on concrete slabs on grade in practice clearly indicate that the drying and shrinkage of concrete slabs on grade play a major role in their overall behaviour. The specific documents dealing with the design of concrete slabs on grade recognise these effects but only give rude methods to take these effects into account.

2.1 Drying of concrete

In order to model the shrinkage behaviour of a concrete element its moisture distribution should be known as a function of time. The moisture transport in a concrete element can be modelled by means of a diffusion model [3]. Often the equations of the diffusion model are written down as a function of the water content w, being the mass of water in a unit volume of concrete. However in this case it is better to use the relative pore humidity H_p to write down the diffusion equations. In the case of variable but uniform temperature T, the diffusion equation becomes

$$\frac{\partial H_p}{\partial t} = div\left(D \; grad \; H_p\right) + \frac{\partial H_s}{\partial t} + K \frac{\partial T}{\partial t} \qquad (1)$$

The last term on the right hand side can be neglected if temperature changes are small. The second term on the right hand side accounts for the change in pore humidity due to the hydration of the cement. For the normal strength concrete used for slabs on grade this term can also be neglected. If the diffusion equation would have been written as a function of the water content w, then the effect of the hydration reaction on the water content could not be neglected. It should also be noted that, due to the hydration reaction, there exists no unique relationship between the relative pore humidity H_p and the water content w of the concrete. Another advantage of using the relative pore humidity lies in the fact that the boundary conditions are often expressed as a function of the relative humidity of the environment H_o (Eq. (2)) and also shrinkage models often use the relative humidity as parameter.

$$d\,\vec{n} \cdot grad \; H_p = \ln\left(\frac{H_p}{H_o}\right) \qquad (2)$$

For a long time the diffusion coefficient D was taken as a constant. It was found however that moisture diffusion in concrete slows down with a factor 10 to 20 once the relative pore humidity drops under a certain level. Several researchers have elaborated equations linking the value of the diffusion coefficient D to the relative pore humidity H_p.

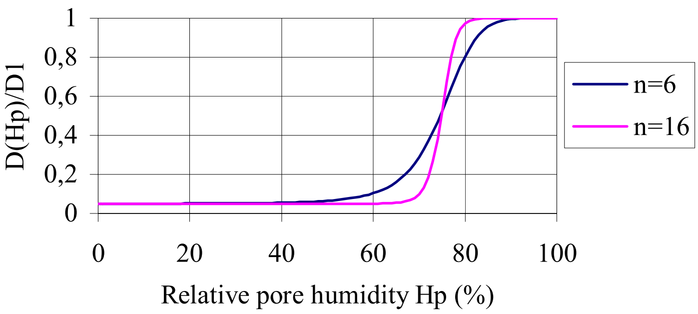

Fig. 5 Diffusion coefficient as a function of relative pore humidity [3]

The relation that is given by Bazant [3] is shown graphically in figure 5. The parameter n determines the width of the region were the change of the diffusion coefficient takes place and D1 represents the diffusion coefficient at a relative pore humidity of 100%. Wong, Wee and Swaddiwudhipong [4] use the same mathematical expression as Bazant but the parameters they use in the expression are dependent on the temperature and the water-cement ratio of the concrete.

By incorporating this diffusion model in a finite element program it is possible to calculate the distribution of the relative pore humidity as a function of time for the entire concrete slab on grade.

2.2 Shrinkage of concrete

The models for the prediction of shrinkage deformations that can be found in codes and regulations only give an average shrinkage value as a function of time for a specific concrete element. This means that only average uniform deformations are estimated. In the Belgian standard NBN B15-002 [5] the shrinkage model is a product of two factors. The first factor determines the ultimate shrinkage deformation for a drying time approaching infinity while the second factor determines the time dependency of the shrinkage deformation. The shrinkage deformation at a time t is then given by

$$\varepsilon_{cs}(t-t_s) = \varepsilon_{cs0} \cdot \beta_s(t-t_s) \tag{3}$$

The ultimate shrinkage deformation is calculated as

$$\varepsilon_{cs0} = \varepsilon_s(f_{cm}) \cdot \beta_{RH} \tag{4}$$

with

$$\varepsilon_s(f_{cm}) = \left[160 + \beta_{sc} \cdot (90 - f_{cm})\right] \cdot 10^{-6} \tag{5}$$

a factor taking into account the effect of the concrete quality on the shrinkage behaviour, and with

$\beta_{RH} = 0.25$ when $RH \geq 99\%$, or $\beta_{RH} = -1.55 \cdot \beta_{sRH}$ when $40\% \leq RH \leq 99\%$.

Hereby β_{sRH} is a coefficient taking into account the influence of the relative humidity *RH* of the environment (in %).

$$\beta_{sRH} = 1 - \left(\frac{RH}{100}\right)^3 \tag{6}$$

The factor determining the time dependency of the deformation is given by

$$\beta_s(t-t_s) = \left[\frac{t-t_s}{0.035 \cdot h_0^2 + t - t_s}\right]^{0.5} \tag{7}$$

with h_0 a shape factor (in mm) given by

$$h_0 = \frac{2 \cdot A_c}{u} \tag{8}$$

with A_c the surface area of a cross section from the concrete element considered and u the perimeter of the concrete element in contact with the environment.

To be able to model the real non-uniform shrinkage behaviour of a concrete slab on grade it is necessary to know at any time t the free shrinkage behaviour in any point of the slab. The free shrinkage is the shrinkage that an elementary volume of concrete would undergo if it were not connected to any neighbouring elementary volumes restraining it. By using the known relative pore humidity distribution and parts of the shrinkage model mentioned in the Belgian code it is possible to calculate the free shrinkage of any point in a concrete slab on grade as a function of time. The time dependency factor given by Eq. (7) is no longer required because the time effect is incorporated in the moisture distribution calculation. The formula for calculating the ultimate shrinkage deformation, given by Eq. (4), can be used to calculate the free shrinkage deformation corresponding to the relative pore humidity H_p in a certain point of the concrete slab. This can be done by replacing the relative humidity of the environment RH in Eq. (6) by the relative pore humidity H_p in the specific point. Instead of calculating one average uniform shrinkage deformation for the entire slab as is done in the code, we now calculate free shrinkage curves for all points in the slab separately. Seen the number of points where this calculation has to be performed it seems obvious to do the calculation by means of a finite element program.

What is actually needed in order to calculate the free shrinkage deformations in a concrete element, caused by a drop in relative pore humidity, is a coefficient that gives the free concrete deformation for a unit change in relative pore humidity and this for any value of the relative pore humidity. This coefficient which can be called a hygroscopic shrinkage coefficient $\alpha(H_p)$ is similar to the thermal expansion coefficient $\alpha(T)$ that gives the volume change of a material for a unit change in temperature in a heat flow problem. Just like the thermal expansion coefficient is dependent on the temperature, the hygroscopic shrinkage coefficient is dependent on the relative pore humidity. The hygroscopic shrinkage coefficient as a function of relative pore humidity can be found by differentiating the expression that gives the shrinkage deformation as a function of the relative pore humidity to this relative pore humidity. By using Eq. (4) and Eq. (6) with RH replaced by H_p we finally obtain

$$\alpha(H_p) = \frac{\partial \varepsilon_{cs}}{\partial H_p} = \varepsilon_s(f_{cm}) \cdot \frac{\partial \beta_{H_p}}{\partial H_p} \tag{9}$$

$$\frac{\partial \beta_{H_p}}{\partial H_p} = 3 \cdot \frac{1.55}{100} \cdot \left(\frac{H_p}{100}\right)^2 \tag{10}$$

In a finite element calculation the relative pore humidity distribution in the concrete slab will be calculated for consecutive time steps Δt starting at time t=0 just after casting. By means of this relative pore humidity distribution the shrinkage deformations in each point of the slab are calculated for the same time steps. The shrinkage deformation in a point at time t_i is calculated starting from the shrinkage deformation in that point at time t_{i-1} by adding the extra shrinkage deformation due to a change in relative pore humidity distribution in that point between times t_{i-1} and t_i. The extra shrinkage deformation for that point is obtained by integrating the hygroscopic shrinkage coefficient between the two values of the relative pore humidity in that point at t_{i-1} and t_i respectively. Written as equation we have

$$\varepsilon_{cs}(t_i) = \varepsilon_{cs}(t_{i-1}) + \Delta\varepsilon_{cs}(t_{i-1}, t_i) \tag{11}$$

$$\Delta\varepsilon_{cs}(t_{i-1}, t_i) = \int_{H_p(t_{i-1})}^{H_p(t_i)} \alpha\big(H_p(t)\big) \cdot dH_p(t) \tag{12}$$

If the calculations were done by hand the free shrinkage deformations as a function of time in each point of the concrete slab on grade would now have been obtained. To find the real deformations the continuity of the concrete material and the boundary conditions of the slab still have to be taken into account. However if the shrinkage model is incorporated in a finite element program, the program automatically takes care of the continuity equations and of the boundary conditions and real deformations are obtained as a calculation result.

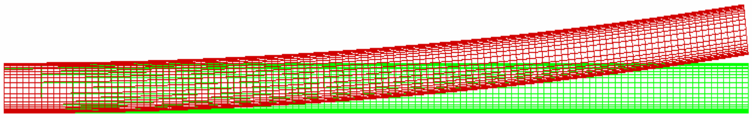

Fig. 6 Curling at one end of a slab in a wind tunnel - finite element calculation

The diffusion model and the shrinkage model were implemented in the finite element program DIANA to check whether the deformation behaviour of the elements used in the experimental test program discussed above can be simulated. Figure 6 shows the curling that was calculated at one end of a slab that was tested in the wind tunnel.

The vertical displacements shown in the figure have been drawn on a different scale than the linear dimensions of the slab.

2.3 Calculation of stresses in the concrete

A further step in the modelling of concrete slabs on grade is the calculation of the stresses in the slab due to the drying and shrinkage of the concrete. Because the slab is studied immediately after casting, the time dependency of the mechanical properties of the concrete has to be considered.

The fast stiffness development of the concrete in combination with important moisture gradients and accompanying hindered shrinkage deformations near the drying surfaces of a concrete slab leads to high tensile stresses at those drying surfaces of the slab in a finite element calculation. The calculated tensile stresses are mostly much higher than the available tensile strength.

In reality however, the tensile stresses near the drying surfaces are much lower. This is due to the time dependent behaviour of the concrete, namely creep and relaxation. To obtain more realistic values for the tensile stresses in a finite element calculation, creep and relaxation models have to be incorporated in the calculation method and this in combination with the time dependent mechanical properties of the concrete.

Basic and drying creep tests are undertaken on young hardening concrete. The results of these tests should allow elaborating a creep and relaxation model for young hardening concrete.

3. Conclusions

The codes and regulations dealing with structural concrete elements also should include slabs on grade. The specific documents that are dealing with slabs on grade could be improved as far as the topic of drying and shrinkage of concrete slabs is concerned. The experimental test program clearly showed the importance of the latter effects. By means of a finite element program including a moisture diffusion model with a shrinkage model coupled to it the deformation behaviour of drying concrete slabs on grade can be modelled in a realistic way. The elaboration of a creep and relaxation model for young hardening concrete should allow the calculation of realistic stresses in the concrete slab.

4. References

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